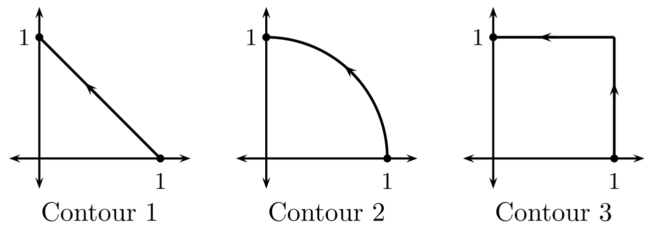
Math 411 Assignment 9 **Due at 3 PM Monday, January 27th**

For this assignment you will need the three contours shown to the right.



When asked to "discuss" results or observations, write at least one complete sentence.

1. (a) Give the standard parametrization of Contour 1. (Remember that your parametrization should include the function $\gamma : \mathbb{R} \to \mathbb{C}$ and a range of values for the parameter t.)
   (b) Compute $\int_\gamma z^2\,dz$ for that contour, showing all steps in the manner done in class Friday. This is a bit tedious, but not as bad as it could be!
   (c) "Feed" your integral of the form $\int_a^b f[\gamma(t)]\,\gamma'(t)\,dt$ prior to any simplification to the Wolfram Definite Integral Calculator. You can find links to it at either 1/24 on the schedule page or on the main Math 411 page. *If you get a different result than you got in (a), make the needed correction(s)!*

2. Contour 1 can also be parameterized by $\gamma(t) = (1 - t^2) + t^2 i$, $0 \le t \le 1$. Give the integral of the form $\int_a^b f[\gamma(t)]\,\gamma'(t)\,dt$ for the same function, $f(z) = z^2$, but using this new parametrization. Then compute (and give!) its value using the Wolfram calculator again. What do you conjecture at this point?

3. (a) Give a parametrization of Contour 2, in terms of the parameter θ.
   (b) Compute $\int_\gamma z^2\,dz$ for Contour 2, working in exponential form. You may wish to refer to Example 1 on page 95 of Churchill and Brown.
   (c) Check your answer using the Wolfram calculator; you might want to use some other variable like x in place of θ.
   (d) Now what are you starting to think? You should have a new conjecture, beyond/different from what you gave in Exercise 2.

4. Now consider Contour 3.
   (a) Give separate parameterizations of the two "legs" of the contour. It is OK to let the parameter go from zero to one for both of them. Denote them by γ_1 and γ_2.
   (b) Note that in this case $\int_\gamma f(z)\,dz = \int_{\gamma_1} f(z)\,dz + \int_{\gamma_2} f(z)\,dz$. Give the sum of integrals with respect to t used to compute this integral.
   (c) Compute the two integrals using the Wolfram calculator and give the value of the integral over the entire contour. Does it support your conjecture from Exercise 3?

5. You will now consider the function $g(z) = |z|^2$. Note that we have an alternate form for $|z|^2$. (definition/theorme sheets...)
   (a) Set up the integral with respect to t for this function over Contour 1 (give the integral of course!) and evaluate it with Wolfram's help.
   (b) Now set up the integral of g over Contour 2 using your parametrization from Exercise 3. Note that you should be able to figure out $|z|^2$ for any z on Contour 2 without any calculations!
   (c) Compute the integral from (b) by hand, then check your answer with Wolfram.
   (d) Consider your results from (a) and (c). What are you thinking?

6. Apply the Cauchy-Riemann equations to the function $g(z) = |z|^2$. What do you conclude?