W14

Math 411, Assignment 9

10 points

(1) a) $\gamma(t) = (1-t) + ti \quad , \quad 0 \leq t \leq 1$ $+\frac{1}{2}$

b) $$\int_\gamma z^2 dz = \int_0^1 [\gamma(t)]^2 \gamma'(t)\,dt$$

$$= \int_0^1 [(1-t)+ti]^2(-1+i)\,dt$$

$$= \int_0^1 [1-2t+\cancel{t^2}-\cancel{t^2}+2t(1-t)i](-1+i)\,dt$$

$+1\frac{1}{2}$

$$= \int_0^1 [(1-2t)+(2t-2t^2)i](-1+i)\,dt$$

$$= \int_0^1 [-1+2t-(2t-2t^2)i+(1-2t)i-(2t-2t^2)]\,dt$$

$$= \int_0^1 [(2t^2-1)+(2t^2-4t+1)i]\,dt$$

$$= \left[\left(\tfrac{2}{3}t^3-t\right)+\left(\tfrac{2}{3}t^3-2t+t\right)i\right]_0^1$$

$$= \boxed{-\frac{1}{3}-\frac{1}{3}i}$$

(2) $$\int_\gamma f(z)\,dz = \int_0^1 [(1-t^2)+t^2 i]^2(-2t+2ti)\,dt = -\frac{1}{3}-\frac{1}{3}i$$ $+\frac{1}{2}$

$+\frac{1}{2}$ The value of $\int_\gamma z^2 dz$ seems to be independent of how γ is parametrized.

③ a) $\gamma(\theta) = e^{i\theta}, \ 0 \le \theta \le \frac{\pi}{2}$ $+\frac{1}{2}$

b) $\int_\gamma z^2 dz = \int_0^{\frac{\pi}{2}} (e^{i\theta})^2 ie^{i\theta} d\theta = i\int_0^{\frac{\pi}{2}} e^{3i\theta} d\theta =$

$+1$

$= \frac{1}{3} e^{3i\theta} \Big|_0^{\frac{\pi}{2}} = \frac{1}{3}\left[e^{\frac{3\pi}{2}i} - e^0\right] = -\frac{1}{3} - \frac{1}{3}i$

$+\frac{1}{2}$ d) The integral $\int_\gamma z^2 dz$ on a contour from $z=1$ to $z=i$ seems to be independent of the contour.

★ ④ a) $\gamma_1(t) = 1 + ti, \ 0 \le t \le 1 \qquad \gamma_2(t) = (1-t) + i, \ 0 \le t \le 1$ $+\frac{1}{2}$

b) $\int_\gamma z^2 dz = \int_{\gamma_1} z^2 dz + \int_{\gamma_2} z^2 dz$

$+1$

$= \int_0^1 [1+ti]^2 i \, dt + \int_0^1 [(1-t)+i]^2 (-1) dt$

$= -\frac{1}{3} - \frac{1}{3}i$

c) It still appears that the value of the integral is independent of the contour from $z=1$ to $z=i$ $+\frac{1}{2}$

Note that $f(z)$ is still z^2 — I forgot to say that on the assignment.

(5) a) $g(z) = |z|^2 = z\bar{z} = x^2 + y^2$, $\gamma(t) = (1-t) + ti$, $0 \le t \le 1$

$+\frac{1}{2}$ $$\int_\gamma z\bar{z}\,dz = \int_0^1 [(1-t)^2 + t^2](-1+i)\,dt = -\frac{2}{3} + \frac{2}{3}i$$

b) $$\int_\gamma z\bar{z}\,dz = \int_0^{\frac{\pi}{2}} e^{i\theta} e^{-i\theta} i e^{i\theta}\,d\theta = \int_0^{\frac{\pi}{2}} i e^{i\theta}\,d\theta = e^{i\theta}\Big|_0^{\frac{\pi}{2}} = -1 + i$$

$\checkmark 1$

$+\frac{1}{2}$ c) In this case the value of the integral from $z = 1$ to $z = i$ depends on the contour.

(6) $g(z) = x^2 + y^2$ $\quad u(x,y) = x^2 + y^2$, $v(x,y) = 0$

$$\frac{\partial u}{\partial x} = 2x, \quad \frac{\partial v}{\partial y} = 0 \implies x = 0$$

$+1$ $$\frac{\partial u}{\partial y} = 2y, \quad \frac{\partial v}{\partial x} = 0 \implies y = 0$$

g is differentiable only at $z = 0$.