$$f'(z_0) = \lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0}$$

$f(z) = z^2$, $z_0 = 1+3i$

$z = 2+2i \to 3+5i$

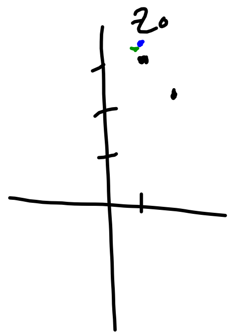
$z = 1.1+3.1i \to 2+6.1i$

$z = 0.99+3.01i \to 1.99+6.01i$

$z = 1.02+2.99i \to 2.02+5.99i$

$$f(z) = z^2$$

$$f'(1+3i) \stackrel{?}{=} 2+6i$$

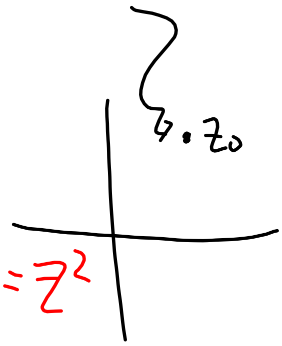


$$f'(z_0) = \lim_{z \to z_0} \frac{z^2 - z_0^2}{z - z_0}$$

$$= \lim_{z \to z_0} \frac{(z+z_0)\cancel{(z-z_0)}}{\cancel{z-z_0}}$$

$$= \lim_{z \to z_0} (z+z_0)$$

$$= 2z_0$$



$$f(z) = z^2$$

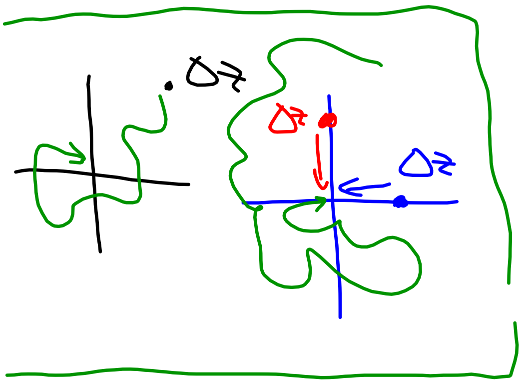
$$f'(z) = 2z$$

$$\lim_{z \to z_0} \frac{f(z)-f(z_0)}{z-z_0} = \lim_{\Delta z \to 0} \frac{f(z_0+\Delta z)-f(z_0)}{\Delta z}$$

If $f(z) = |z|^2$ differentiable anywhere?

$$\frac{f(z_0+\Delta z)-f(z_0)}{\Delta z} = \frac{|z_0+\Delta z|^2 - |z_0|^2}{\Delta z}$$

$$= \frac{(z_0+\Delta z)\overline{(z_0+\Delta z)} - z_0\bar{z}_0}{\Delta z}$$



$$= \frac{\cancel{z_0\bar{z}_0} + \bar{z}_0\Delta z + z_0\overline{\Delta z} + \Delta z\overline{\Delta z} \cancel{- z_0\bar{z}_0}}{\Delta z}$$

$$= \bar{z}_0 + \overline{\Delta z} + z_0\frac{\overline{\Delta z}}{\Delta z}$$

$$\lim_{\Delta z \to 0}\left(\bar{z}_0 + \cancelto{0}{\overline{\Delta z}} + z_0\frac{\overline{\Delta z}}{\Delta z}\right) = \bar{z}_0 + z_0$$

$$= \bar{z}_0 - z_0$$

equal?

Only if $z_0 = 0$

Many derivatives are same as in $\mathbb{R}$: $\frac{d}{dz}c = 0$ c is a constant, possibly complex

$f(z)=c$

$\frac{d}{dz}z = 1$

$\frac{d}{dz}[cf(\cancel{z})] = c\frac{d}{d\cancel{z}}f(x) = cf'(x)$

$\frac{d}{dz}[f(z)+g(z)] = \frac{d}{dz}f(z)+\frac{d}{dz}g(z)$

$\frac{d}{dz}$ is a linear operator

$f(z) = \frac{3z+1}{z-i}$

$\overline{z-(2-i)}$

$\frac{d}{dz}z^n = nz^{n-1}$

Prod + Quotient rule

$f(z) = (5+2i)z^3 - 7z^2 + (4-3i)$

What about $f(z) = x^2 + i y^2$? $z = x + iy$

Cauchy-Riemann Equations

If $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ at $z_0 = x_0 + iy_0$,

$z = x + iy$ then f is diff there and

$f(z) = w = u + iv$ $f'(z_0) = u_x(z_0) + i v_x(z_0)$

$f(z) = x^2 + iy^2$

$u(x,y) = x^2$, $v(x,y) = y^2$

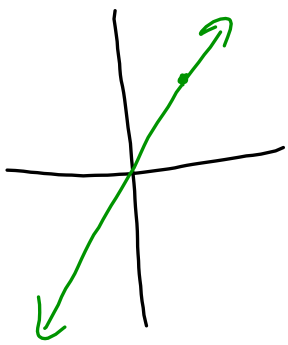
$\frac{\partial u}{\partial x} = 2x$ $\quad \frac{\partial u}{\partial y} = 0$ $\quad \frac{\partial v}{\partial x} = 0$ $\quad \frac{\partial v}{\partial y} = 2y$

$2x = 2y$ by first C-R eqn

$0 = -0$ by second C-R eqn

For what x+y values are these true?

$x = y$



$f'(z) = u_x + iv_x$

$= 2x$

Assn 5: 4G: pg. 26

12 b, f, g, h

Give u, v, partials
tell where diff
tell derivative