12 b) $f(z) = 2x + ixy^2$

$u(x,y) = 2x \quad , v(x,y) = xy^2$

$\frac{\partial u}{\partial x} = 2 \quad , \frac{\partial v}{\partial y} = 2xy \implies 2 = 2xy \implies 1 = xy$

$u_y(x,y) = 0 \, , v_x(x,y) = y^2 \implies 0 = -y^2 \implies y = 0$

But $xy \neq 1$ if $y = 0$, so f is not differentiable.

12) h) $f(z) = z \operatorname{Im} z = (x+iy)y = xy + iy^2$

$u(x,y) = xy$, $v(x,y) = y^2$

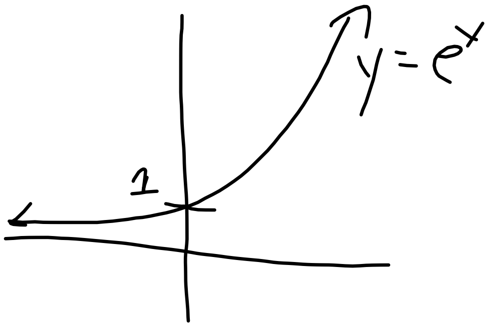
$\frac{\partial u}{\partial x} = y$, $\frac{\partial v}{\partial y} = 2y \implies y = 2y \implies 0 = y$

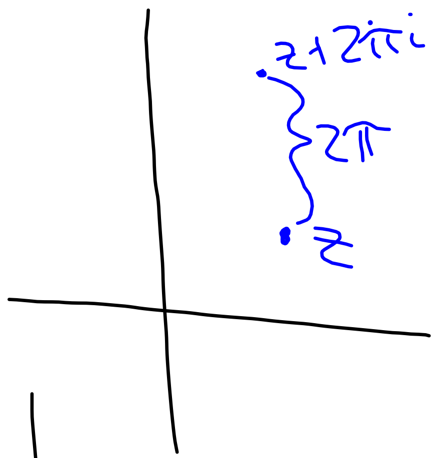
$\frac{\partial u}{\partial y} = x$, $\frac{\partial v}{\partial x} = 0 \implies x = 0$

f is differentiable at $z = 0$.

$f'(z) = y + 0i$, so $f'(0) = 0$.

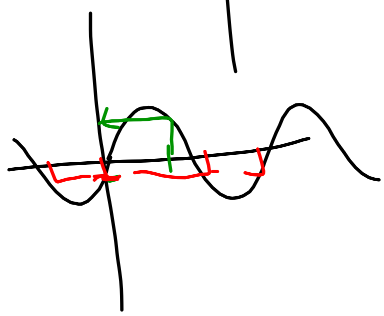
$$e^{z} = \exp(z) = e^{x}(\cos y + i \sin y) = e^{x}e^{iy}$$

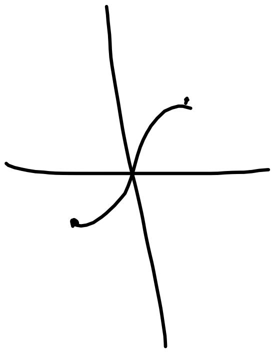




$e^{x} > 0$

$e^{z} > 0$





CB, pg. 66

1a) Show that $\exp(2 \pm 3\pi i) = -e^2$

$$\exp(2 \pm 3\pi i) = \exp(\underbrace{2 + \pi i}_{z} + 2\pi i)$$

$$= \exp(2 + \pi i) \quad \text{Lemma 3.16(c)}$$

$$= \exp(2) \cdot \exp(\pi i) \quad \text{Lemma 3.16(a)}$$

$$= e^2(-1)$$

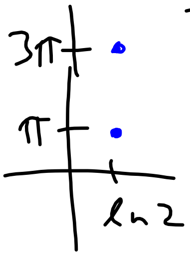
$$= -e^2$$

$$\exp(2 \pm 3\pi i) = e^2\big(\underbrace{\cos(\pm 3\pi)}_{-1} + \underbrace{i\sin(\pm 3\pi)}_{0}\big)$$

$$= -e^2$$

$$\{[(\ )]\}$$

3a) Find all values of z such that $e^z = -2$



$$e^x e^{iy} = 2e^{i(\pi+2\pi n)}$$

$$e^x = 2$$

$$x = \ln 2$$

$$e^{iy} = e^{i(\pi+2\pi n)}$$

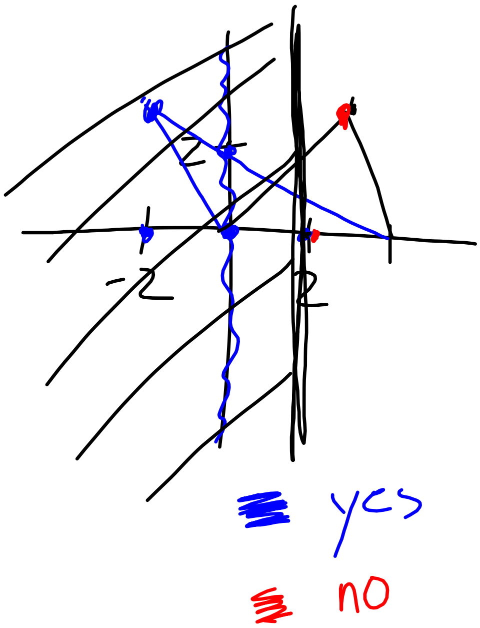
$$y = \pi + 2\pi n$$

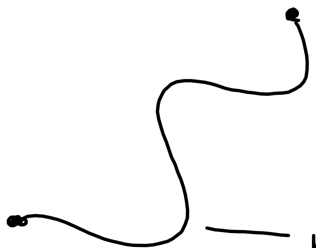
$$z = \ln 2 + i(\pi + 2\pi n)$$

$$\underbrace{|z-4|}_{\text{Distance from } z \text{ to } 4} \geq |z|$$

$|z-0|$

Distance from z to 4





path, arc, curve
(smooth)