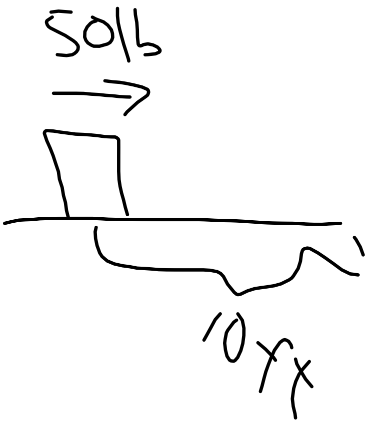
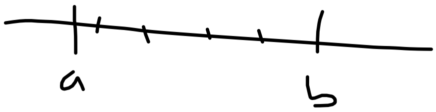
Integration

$W = Fd$ $W = PV$

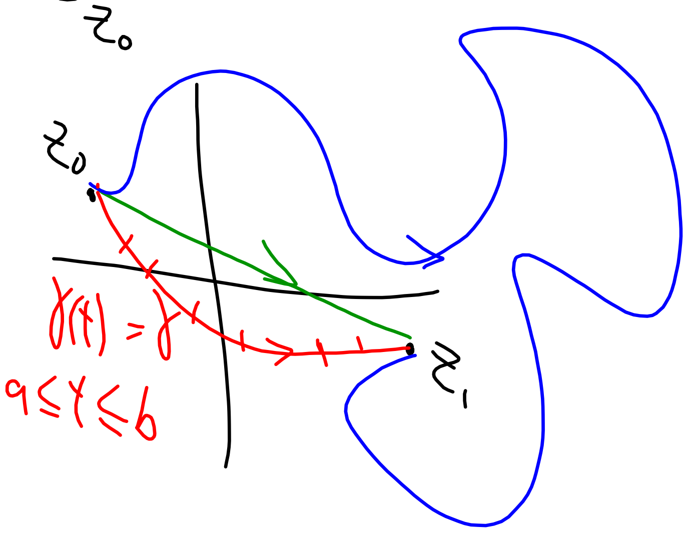
$F = ma$ $A = lw$



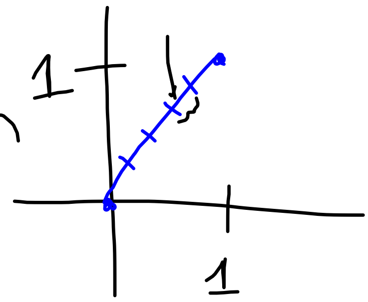
$$\int_a^b f(x)\,dx = \lim_{?} \sum f(x_i)\Delta x_i$$



$$\int_{z_0}^{z_1} f(z)\,dz = \lim \sum f(z)\Delta z$$



Integrate $f(z)=z^2$ on



$$\int_0^{1+i} f(z)\,dz \approx \sum_{k=1}^{5} f(z_k)\,\Delta z_k$$

k	z_k	$f(z_k)$	Δz_k	$f(z_k)\Delta z_k$
1			$.2+.2i$	$-.004+.004i$
2	$.3+.3i$	$.18i$	"	$-.036+.036i$
3	$.5+.5i$	$.5i$	"	$-.1+.1i$
4			"	$-.196+.196i$
5	$.9+.9i$	$1.62i$	$.2+.2i$	$-.324+.324i$
				$-.66+.66i$

Evaluate $\int_0^{1+i} z^2 \, dz$ on the path

$\gamma(t) = t^2 + t^2 i, \quad 0 \le t \le 1$

$\gamma'(t) = \frac{d\gamma}{dt} = 1 + i$

$f(\gamma(t))$

$$\int_0^{1+i} z^2 \, dz = \int_0^{1+i} [\gamma(t)]^2 \, d\gamma = \int_{\gamma=0}^{\gamma=1+i} [\gamma(t)]^2 \frac{d\gamma}{dt} \, dt$$

$$= \int_{t=0}^{t=1} (t + ti)^2 (1+i) \, dt$$

$(t+ti)^2 = 2t^2 i$

$(t+ti)^2(1+i)$

$= 2t^2 i(1+i)$

$= -2t^2 + 2t^2 i$

$$= \int_0^1 (-2t^2 + 2t^2 i) \, dt$$

$$= -\frac{2}{3}t^3 + \frac{2}{3}t^3 i \Big]_0^1$$

$$= -\frac{2}{3} + \frac{2}{3} i$$

$$\operatorname{Re}(z^2) > 0$$

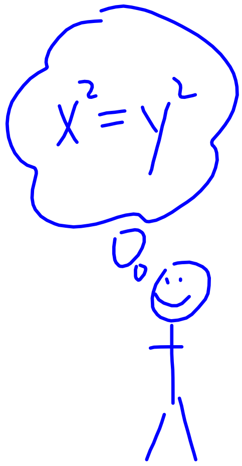
$$z^2 = (x+iy)^2$$

$$= x^2 + 2xyi - y^2$$

$$\operatorname{Re}(z^2) = x^2 - y^2$$

$$x^2 - y^2 > 0$$

$$x^2 > y^2$$



$$\sqrt{x^2} = |x|$$