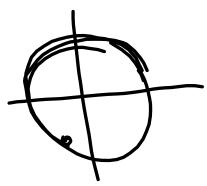
Find all z such that $e^z = 1+\sqrt{3}\,i$

$$e^z = e^{x+iy} = e^x e^{iy}$$
$$= e^x(\cos y + i\sin y)$$
$$= e^x\cos y + (e^x \sin y)i$$

$$z = \ln 2 + \left(\frac{\pi}{3} + n\pi\right)i$$



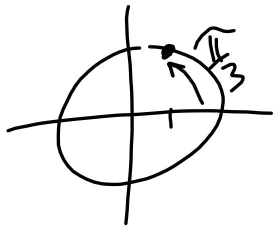
$$n=1 \Rightarrow z = \ln 2 + \frac{4\pi}{3}i$$

$$e^z = e^{\ln 2 + \frac{4\pi}{3}i} = e^{\ln 2}\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$$
$$= 2\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = -1-\sqrt{3}\,i$$

$$e^z = 1 + \sqrt{3}\, i$$

$$= 2\left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)$$

$$= 2\left(\cos\frac{\pi}{3} + i \sin\frac{\pi}{3}\right)$$



$$e^x e^{iy} = 2 e^{\left(\frac{\pi}{3} + 2n\pi\right) i}$$

$$e^x = 2 \qquad y = \frac{\pi}{3} + 2n\pi$$

$$x = \ln 2$$

$$\boxed{z = \ln 2 + \left(\frac{\pi}{3} + 2n\pi\right) i}$$

$$\left|\exp(2z+i)+\exp(iz^2)\right|$$

$$\leq \left|\exp(2z+i)\right| + \left|\exp(iz^2)\right|$$

$$=$$

$$=$$

$$= e^{2x} + e^{-2xy}$$

$$S = 1 + r + r^2 + r^3 + r^4 + \cdots$$ converges to $\frac{1}{1-r}$ if $|r|<1$

Geometric series with ratio r.

$$\begin{array}{rl} S_n = & 1 + r + r^2 + r^3 + \cdots + r^n \\ -rS_n & \phantom{1+{}} r + r^2 + r^3 + \cdots + r^n + r^{n+1} \\ \hline \end{array}$$

$$S_n - rS_n = 1 - r^{n+1}$$

$$S_n(1-r) = 1 - r^{n+1}$$

$$S_n = \frac{1-r^{n+1}}{1-r} \xrightarrow[|r|<1]{} \frac{1}{1-r}$$

$$1-\frac{1}{3}x+\frac{1}{9}x^2-\frac{1}{27}x^3+\frac{1}{81}x^4-\dots$$

$r=-\frac{1}{3}x$

Radius of convergence? $\rightarrow$ $\left|-\frac{1}{3}x\right|<1$

$\left|-\frac{1}{3}\right||x|<1$

$\frac{1}{3}|x|<1$

$|x|<3$

Radius of convergence is 3

What does it converges to?

$$\frac{1}{1-r}=\frac{1}{1-(-\frac{1}{3}x)}=\frac{1}{1+\frac{1}{3}x}$$

$$=\frac{3}{3+x}$$

$$\frac{5}{2-x}$$

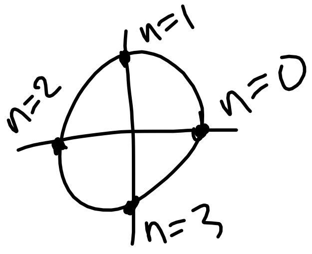
Find a geometric series, give radius of convergence.

$$\frac{5}{2-x} = 5\left(\frac{1}{2-x}\right)$$

$$= \frac{5}{2}\left(\frac{1}{1-\frac{1}{2}x}\right)$$

$\frac{1}{1-r}$ $r=\frac{1}{2}x$

$$= \frac{5}{2}\left(1+\frac{1}{2}x+\frac{1}{4}x^2+\frac{1}{8}x^3+\cdots\right)$$



$\sqrt{4} = 2, -2$

Fourth roots of 1:

$\sqrt[4]{1} = \boxed{i, -1, 1, -i}$ $z = e^{10\pi i}$

$i^4 = (i^2)^2 = 1$

$z = e^{\frac{\pi}{2} n i}$