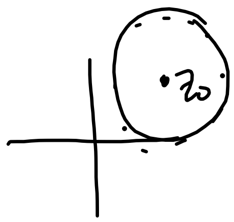
Power series

$$a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$

$$a_0 + a_1(z-z_0) + a_2(z-z_0)^2 + \cdots$$



P.S. "centered" at z_0

Laurent Series centered at z_0

$$\cdots + \frac{a_2}{(z-z_0)^2} + \frac{a_{-1}}{z-z_0} + a_0 + a_1(z-z_0)^1 + a_2(z-z_0)^2 + \cdots$$

Integrating a single term:

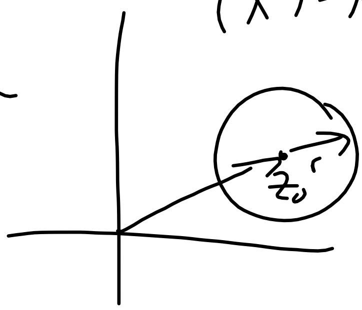
$$\int_{C_r(z_0)} a_n(z-z_0)^n dz = a_n \int_{C_r(z_0)} (z-z_0)^n dz = \begin{cases} 0 \text{ if } n \neq -1 \\ 2\pi i \text{ if } n = -1 \end{cases}$$

a_n

$C_r(z_0) = z_0 + re^{i\theta} = \gamma(\theta)$

$0 \le \theta \le 2\pi$

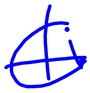
$(x^a)^b = x^{ab}$



$\gamma'(\theta) = rie^{i\theta}$

$$\int_{C_r(z_0)} (z-z_0)^n \, dz = \int_0^{2\pi} \left[(\cancel{z_0} + re^{i\theta}) - \cancel{z_0}\right]^n rie^{i\theta} \, d\theta$$

$$= \int_0^{2\pi} r^n e^{in\theta} rie^{i\theta} \, d\theta$$



$$= r^{n+1} i \int_0^{2\pi} e^{in\theta + i\theta} \, d\theta$$

$$= r^{n+1} i \int_0^{2\pi} e^{(n+1)\theta i} \, d\theta$$

$$= r^{n+1} i \left[\frac{1}{(n+1)i} e^{(n+1)\theta i}\right]_0^{2\pi}$$

$$= \frac{r^{n+1}}{n+1} \left[e^{(n+1)i\theta}\right]_0^{2\pi}$$

$n = -1$

$$= i \int_0^{2\pi} d\theta$$

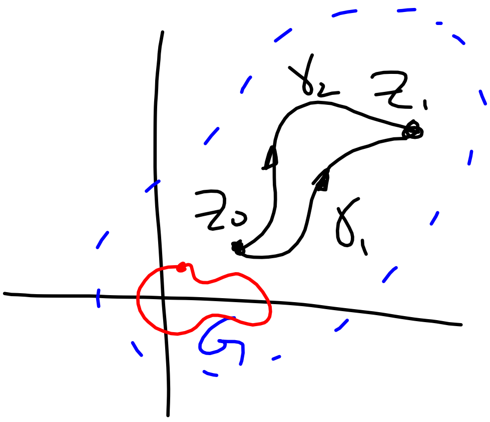
$$= i\theta \Big|_0^{2\pi}$$

$$= 2\pi i$$

$n \ne -1$

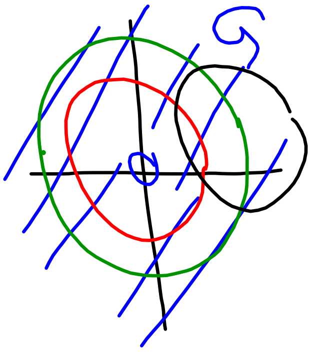
$$\frac{r^{n+1}}{n+1} \left[\cancelto{1}{e^{(n+1)2\pi i}} - \cancelto{1}{e^0}\right]$$

$$= 0$$



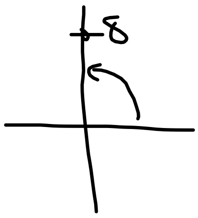
$$\int_\gamma f(z)\,dz$$

$$\int_{C_R(0)} \frac{1}{z}\,dz = 2\pi i \qquad \int_{C_R(0)} \frac{1}{(z-0)}\,dz$$



Find the 3rd roots of $8i$

(1) Write $8i$ in exponential form.



$$8i = 8e^{\left(\frac{\pi}{2}+2\pi n\right)i}$$

$(ab)^n = a^n b^n$

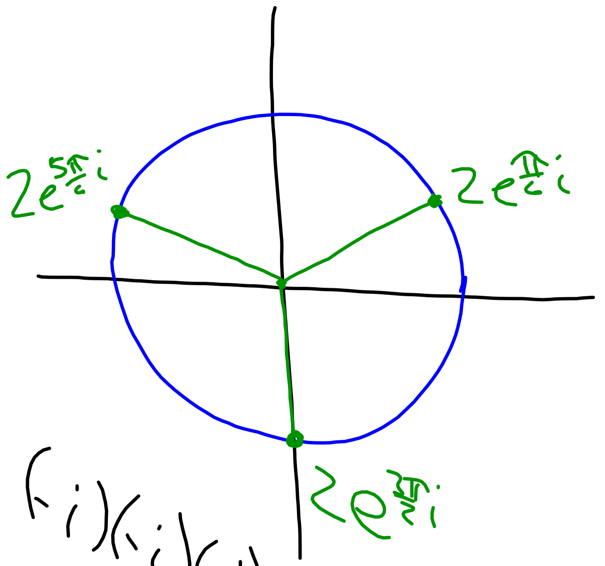
(2) Take root: $\sqrt[3]{8i} = \left(8e^{\left(\frac{\pi}{2}+2\pi n\right)i}\right)^{\frac{1}{3}}$

$$= 2e^{\left(\frac{\pi}{6}+\frac{2\pi}{3}n\right)i}$$

All the 3rd roots of $8i$

$n=0$ gives the principal root: $2e^{\frac{\pi}{6}i}$

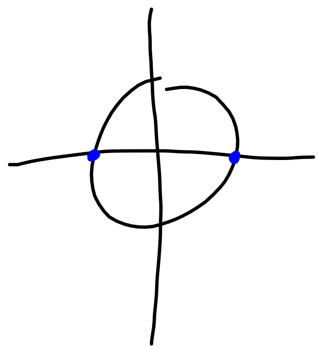
$$\sqrt[3]{8i} = 2e^{\left(\frac{\pi}{6}+\frac{2\pi}{3}n\right)i}$$

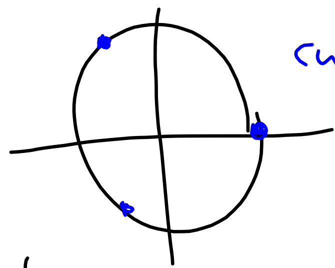


$(-i)(-i)(-i)$

$= -i^3$

$-i^2 \cdot i$





cube roots of unity