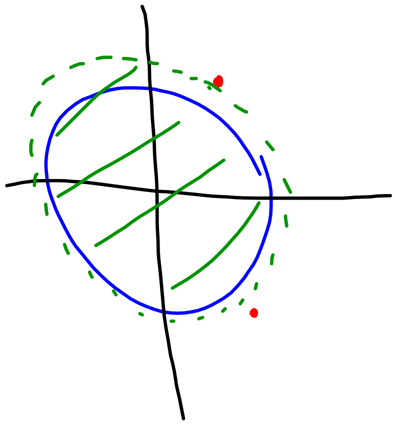
2(a) $\displaystyle\int_{C_2(0)} \frac{z+2}{z^2-2z+5}\,dz$

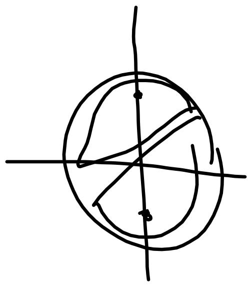
roots $1 \pm 2i$



$G \subseteq \mathbb{C}$

~~$G = \mathbb{C}$~~

$$\#3 \quad \int_{C_3(0)} \frac{1}{(z^2+4)^2}\, dz = \int_{\gamma_1} \frac{1/(z+2i)^2}{(z-2i)^2} \int_{\gamma_2}$$



$=$

$f(z)$

$\sqrt{25} = -5$

$= 5$

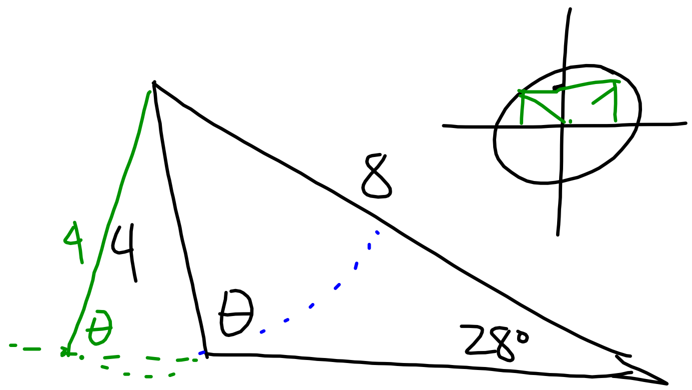
Let's think of $f(x) = \sqrt{x}$ as a multi-valued function.

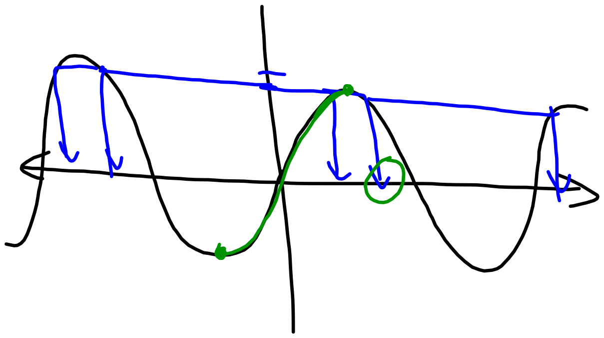
$\sqrt{x}$ is any number that can be squared to get x.

2 poosible ways to make our multi-valued square root single-valued:

$f(x) = \sqrt{x}$ $\quad f(x) \geq 0$ ⟶ Two branches

OR $f(x) = \sqrt{x}$ $\quad f(x) \leq 0$ ⟶ of the square root.





$$\frac{\sin 28^\circ}{4} = \frac{\sin\theta}{8}$$

$$2\sin 28^\circ = \sin\theta$$

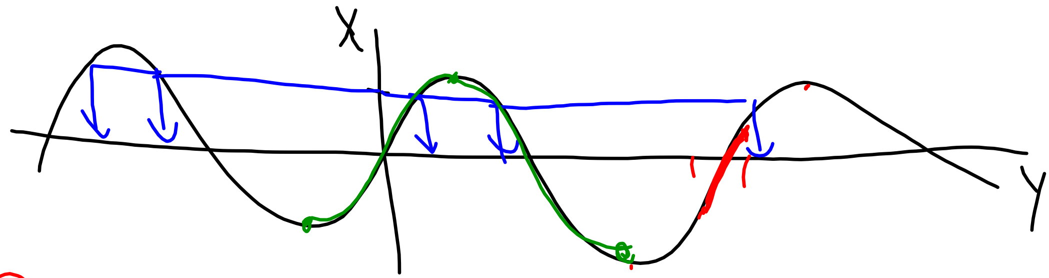
$$0.934 = \sin\theta$$

$$\sin^{-1}(0.934) \approx \theta$$

$$70^\circ \approx \theta$$

$y = \sin^{-1} x$, where $\frac{\pi}{2} \le y \le \frac{3\pi}{2}$

y is any value so that $\sin y = x$



Branches of $\sin^{-1}$:

⋮

$y = \sin^{-1}(x) \quad -\frac{\pi}{2} < y \le \frac{\pi}{2}$ Principal Branch

$y = \sin^{-1}(x) \quad \frac{\pi}{2} < y \le \frac{3\pi}{2}$

⋮

$f(z) = \sqrt{z}$

Branches of $\sqrt{z}$

$f(z) = \sqrt{z}$

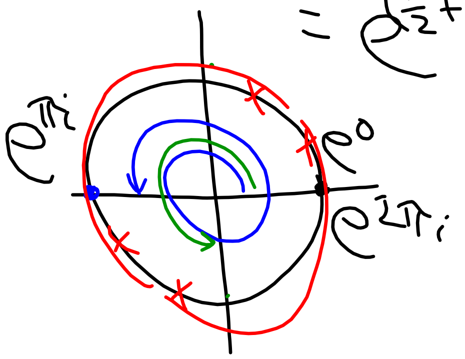
$\sqrt{-1} = \left(1 e^{(\pi + 2\pi n) i}\right)^{\frac{1}{2}}$

$0 \le \arg f(z) < \pi$

or

$\pi \le \arg f(z) < 2\pi$

$= e^{(\frac{\pi}{2} + \pi n) i}$

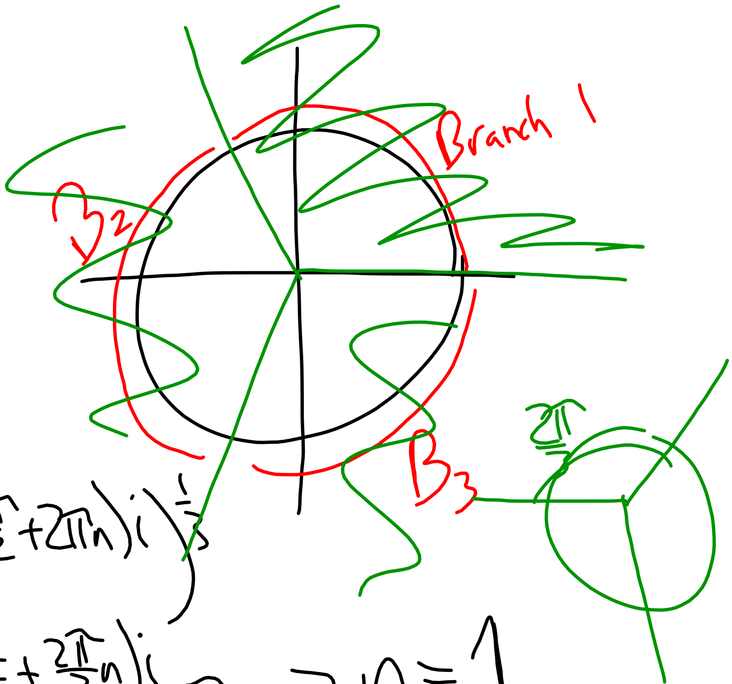


Distinct roots

$e^{\frac{\pi}{2} i} = i, \; e^{\frac{3\pi}{2} i} = -i$

Find $\sqrt[3]{-8i}$

in Branch 2



$$(-8i)^{\frac{1}{3}} = \left(8e^{\left(\frac{3\pi}{2}+2\pi n\right)i}\right)^{\frac{1}{3}}$$

$$= 2e^{\left(\frac{\pi}{2}+\frac{2\pi}{3}n\right)i} \rightsquigarrow n=1$$