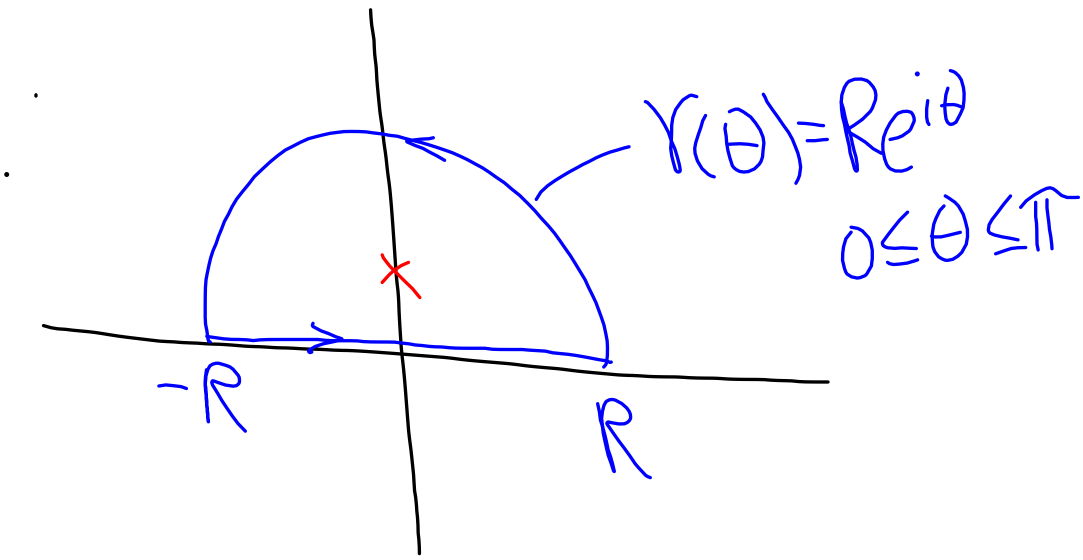
$$\int_{-\infty}^{\infty} f(x)\,dx = \frac{\pi}{e}$$



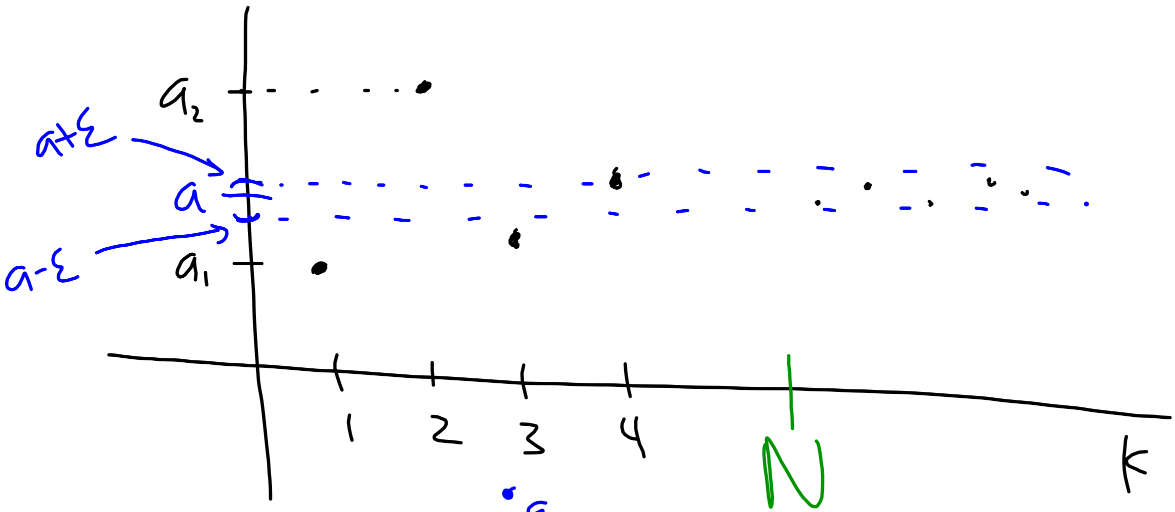
Sequences

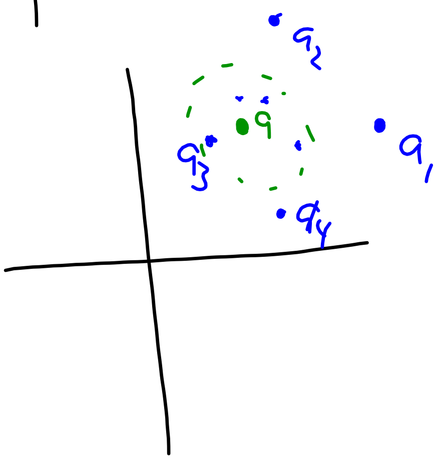
$a_1, a_2, \ldots$ and a all possibly complex.

- $(a_k) = a_1, a_2, a_3, a_4, \ldots$
- Converges if there is a number a such that given an $\varepsilon > 0$, there exists an N such that $|a_k - a| < \varepsilon$ whenever $k \geq N$.

limit

We write $\lim_{k \to \infty} a_k = a$


a_2
$a+\varepsilon$
a
$a-\varepsilon$
a_1
1
2
3
4
N
k


a_2
a
a_1
a_3
a_4

Series

$$a_1+a_2+a_3+\dots=\sum_{k=1}^{\infty} a_k = \sum_{k\geq 1} a_k$$

$$a_1+a_2+a_3+a_4+a_5+\dots$$

$a_1 = S_1$ ← Partial sums

$a_1+a_2 = S_2$ ←

$a_1+a_2+a_3 = S_3$ ←

$\vdots$

The series converges if the sequence of partial sums converges.

$$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots=\sum_{k=0}^{\infty}\frac{1}{2^k}=2$$

Geometric, $r=\frac{1}{2}$

$$1+r+r^2+r^3+\cdots=\frac{1}{1-r} \quad \text{if } |r|<1$$

$$\underbrace{1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots}=\sum_{k=1}^{\infty}\frac{1}{k}$$

$$> \underbrace{\frac{1}{2}}_{\frac{1}{2}}+\underbrace{\frac{1}{2}}_{\frac{1}{2}}+\underbrace{\frac{1}{4}+\frac{1}{4}}_{\frac{1}{2}}+\underbrace{\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}}_{\frac{1}{2}}+\frac{1}{16}+\frac{1}{16} \qquad \frac{1}{16}+\frac{1}{32}$$

Assume $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\dots$ converges,

say to L.

$$L = 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\dots$$

$$> \frac{1}{2}+\frac{1}{2}+\frac{1}{4}+\frac{1}{4}+\frac{1}{6}+\frac{1}{6}+\frac{1}{8}+\frac{1}{8}$$

$$= 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\dots$$

$$= L$$