numerical

Does a series converge?

Geometric Series

$1+r+r^2+r^3+r^4+\ldots$ converges to

$\frac{1}{1-r}$ if $|r|<1$

p-series

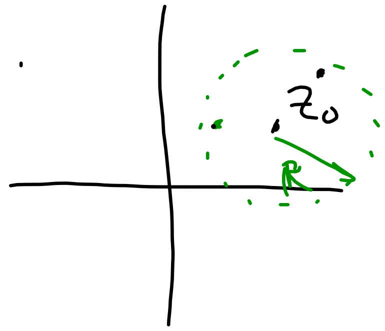
$1+\frac{1}{2^p}+\frac{1}{3^p}+\frac{1}{4^p}+\ldots$ Converges if $p>1$

Power Series

Laurent

$$c_0 + c_1(z-z_0) + c_2(z-z_0)^2 + \cdots$$

$$= \sum_{k=0}^{\infty} c_k (z-z_0)^k$$

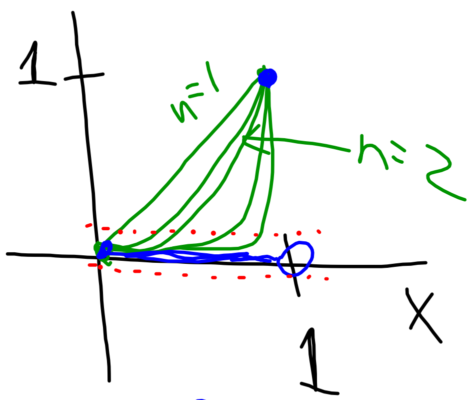


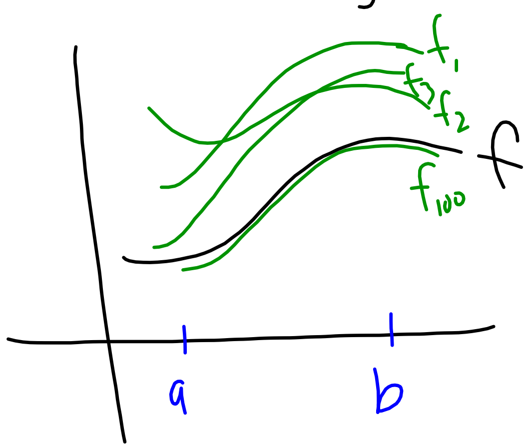
Power series centered at z_0

For what values of z does one of these converge?

$$f_n(x) = x^n \qquad n = 1, 2, 3, 4, \ldots \qquad 0 \le x \le 1$$

Do these functions converge to a function f?

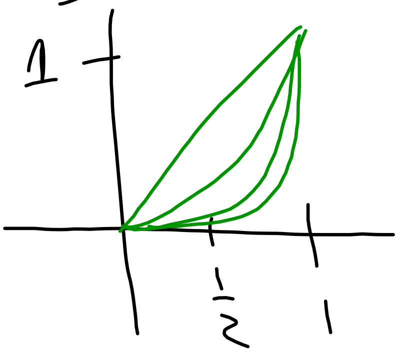




$$f(x) = \begin{cases} 0 & \text{if } 0 \le x < 1 \\ 1 & \text{if } x = 1 \end{cases}$$

Our sequence converges pointwise to this function

$g_n(x) = x^n \quad 0 \le x \le \frac{1}{2}$



converge uniformly to

$g(x) = 0$ on $[0, \frac{1}{2}]$

$1 + z + z^2 + z^3 + \cdots$

$1 + 2 + 3 + \cdots$ numerical

What is R for $\sum c_k (z-z_0)^k$?

$$R = \lim_{k\to\infty} \left| \frac{c_k}{c_{k+1}} \right|$$

$$R = \frac{1}{\lim_{k\to\infty} \sqrt[k]{|c_k|}} = \lim_{k\to\infty} \frac{1}{\sqrt[k]{|c_k|}}$$

$$e^z = 1 + z + \frac{z^2}{2} + \frac{z^3}{3!} + \frac{z^4}{4!} + \cdots$$

$z_0 = 0$

$$c_0 + c_1(z - z_0) + c_2(z - z_0)^2 + \cdots \qquad c_k = \frac{1}{k!}$$

$$R = \lim_{k\to\infty} \frac{1}{|c_k|^{1/k}} = \lim_{k\to\infty} \frac{1}{\sqrt[k]{\frac{1}{k!}}} = ?$$

we think ∞

$$R = \lim_{k\to\infty} \left|\frac{c_k}{c_{k+1}}\right| = \lim_{k\to\infty} \left|\frac{\frac{1}{k!}}{\frac{1}{(k+1)!}}\right| =$$

$$\lim_{k\to\infty} \left|\frac{(k+1)!}{k!}\right| = \lim_{k\to\infty} \frac{(k+1)k(k-1)(k-2)\cdots 1}{k(k-1)(k-2)\cdots 1}$$

$$= \lim_{k\to\infty} (k+1) = \infty$$

$$e^z = 1 + z + \frac{z^2}{2} + \frac{z^3}{3!} + \cdots$$

$$z^4 e^{z^2} = z^4\left(1 + z^2 + \frac{z^4}{2} + \frac{z^6}{3!} + \cdots\right)$$

$$= z^4 + z^6 + \frac{z^8}{2} + \frac{z^{10}}{3!} + \cdots$$

$$\frac{1}{3+2z} = \qquad = \bigcirc\left(\frac{1}{1-\bigcirc}\right)$$

$$|e^{iz}| =$$

$$z = x + iy$$

$$\operatorname{Im} z \geq 0$$

$$\leq B$$

$$y \geq 0$$

Bound