Assn 15

$$|\cos z| = \left| \frac{e^{iz} + e^{-iz}}{2} \right|$$

$$= \frac{|e^{iz} + e^{-iz}|}{|2|}$$

$$\leq \frac{1}{2}\left(|e^{iz}| + |e^{-iz}|\right)$$

$$\leq \frac{1}{2}(1+1)$$

No!

$$= 1$$

$$|\cos z| = \left|\frac{e^{iz}+e^{-iz}}{2}\right|$$

$$= \frac{|e^{iz}+e^{-iz}|}{|2|}$$

$z = x+iy$

$iz = -y+ix$

$-iz = y-ix$

$$\leq \frac{1}{2}\left(|e^{iz}|+|e^{-iz}|\right)$$

$$= \frac{1}{2}\left(|e^{-y+ix}|+|e^{y-ix}|\right)$$

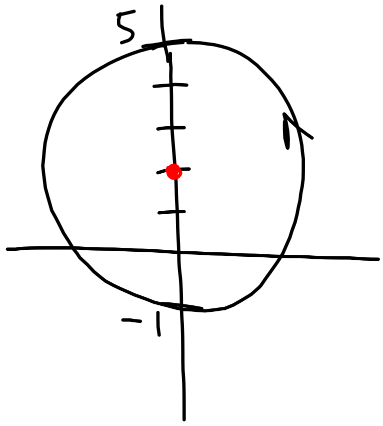
$$= \frac{1}{2}\left(|e^{-y}|\cancelto{1}{|e^{ix}|}+|e^{y}|\cancelto{1}{|e^{-ix}|}\right)$$

$$= \frac{1}{2}\left(e^{-y}+e^{y}\right) = \cosh(y)$$

$$\leq \frac{1}{2}\left(e^{|y|}+e^{|y|}\right)$$

$$= \frac{1}{2}\left(2e^{|y|}\right) = e^{|y|}$$

#2 $\gamma(\theta) = 2i + 3e^{i\theta}, \quad 0 \leq \theta \leq 2\pi$



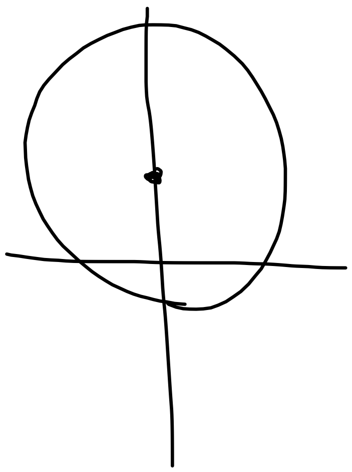
$$|z^2+5z-2| \leq |z^2| + |5z| + |-2|$$

$$= |z|^2 + 5|z| + 2$$

$$\leq 25 + 25 + 2$$

$$= 52$$

b) $$\left|\frac{z^2+5z-2}{z-2i}\right| = \frac{|z^2+5z-2|}{|z-2i|}$$

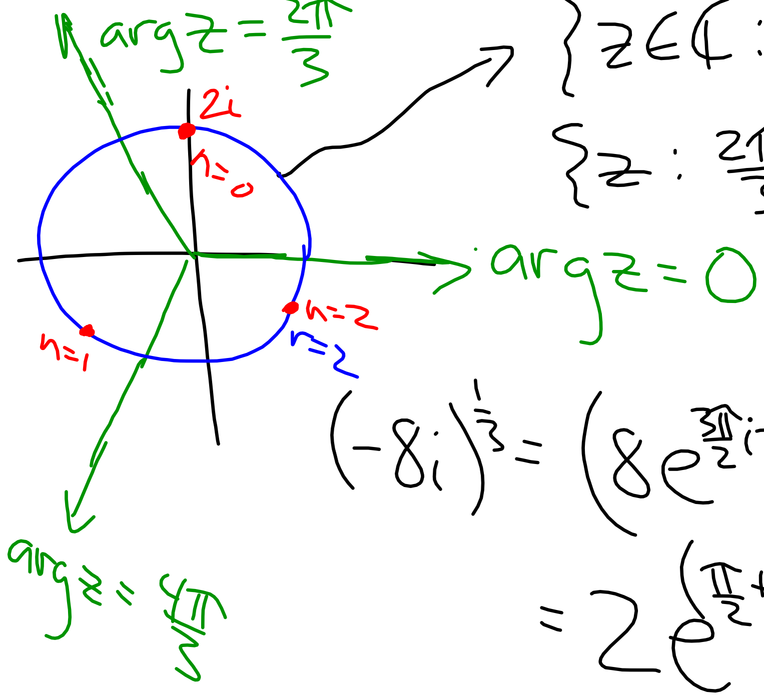


$$\le \frac{52}{|z-2i|}$$

$|z_1 - z_2|$

$$= \frac{52}{3}$$

Assn. 15, #4



$$\{z \in \mathbb{C} : 0 \le \arg z < \tfrac{2\pi}{3}\}$$

$$\{z : \tfrac{2\pi}{3} \le \arg z < \tfrac{4\pi}{3}\}$$

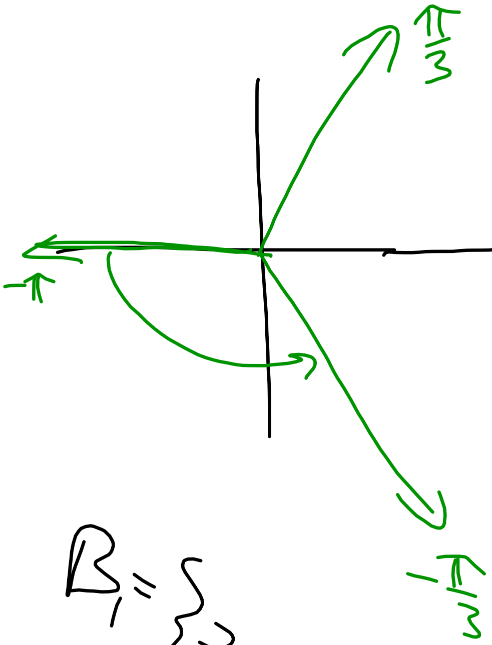
$$(-8i)^{\frac{1}{3}} = \left(8e^{\frac{3\pi}{2}i + 2\pi n i}\right)^{\frac{1}{3}}$$

$$= 2e^{\left(\frac{\pi}{2} + \frac{2\pi}{3}n\right)i}$$

$$n=2: \quad = 2e^{\left(\frac{\pi}{2} + \frac{4\pi}{3}\right)i}$$

$$= 2e^{\frac{11\pi}{6}i}$$

$$= 2\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = \sqrt{3} - i$$



$$B_1 = \{z : -\pi \le \arg z < -\frac{\pi}{3}\}$$

$$B_2 = \{z : -\frac{\pi}{3} \le \arg z < \frac{\pi}{3}\}$$

$$B_3 = \{z : \frac{\pi}{3} \le \arg z < \pi\}$$

$$B_1 = \{z : \theta \le \arg z < \theta + \frac{2\pi}{3}\} = \{re^{i\theta} : \frac{\pi}{3} \le \theta < \pi\}$$

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Thm. 5.13:

$$\int_{\gamma} f(z)\,dz = F(\gamma(b)) - F(\gamma(a))$$

Lemma $\rightarrow$ Thm

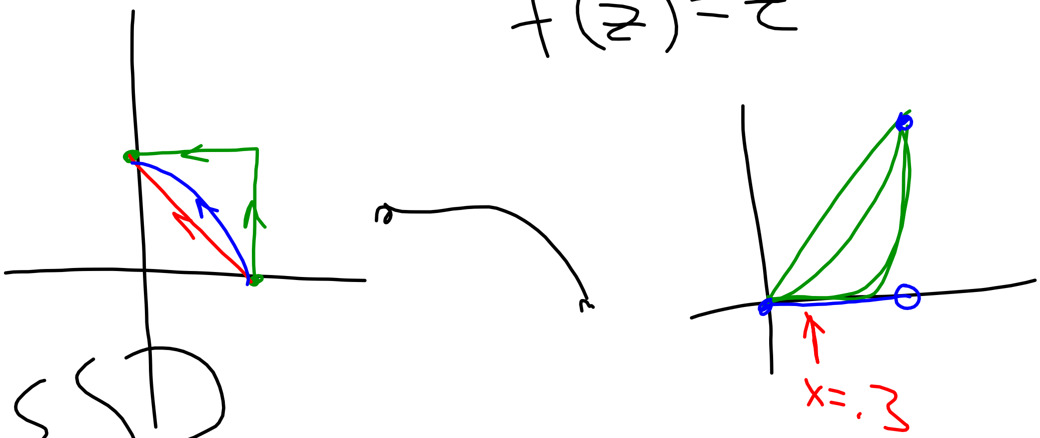
Thm $\rightarrow$ Cor

Thm $\rightarrow$ Cor

Thm $\rightarrow$ General mathematical truth

Prop ?

$$f(z) = z^2$$



$$= \sum$$

$$f(z) = \sim + \sim + \sim + \sim + \sim \cdots$$

$$f'(z) = \sum_{k=1}^{\infty}$$

$$|e^{iz}| = |e^{i(x+iy)}|$$

$$= |e^{ix}||e^{-y}|$$

$$= 1e^{-y} \leq 1$$