$f(z)$
holomorphic
at z_0

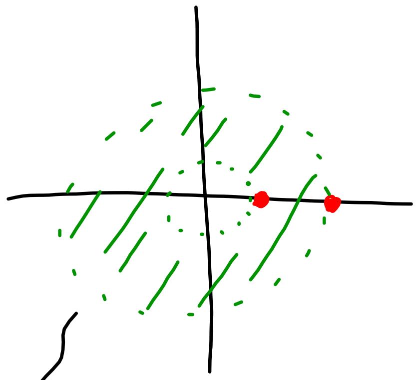
$$f(z) = \sum_{k=0}^{\infty} c_k (z - z_0)^k$$

analytic

f is holomorphic @ z_0. How do we get a power series for it, centered at z_0?

$$\frac{5}{3+z}$$

$$f(z) = \frac{-1}{(z-1)(z-2)}$$



$$\sum_{k=-\infty}^{\infty} c_k (z-z_0)^k$$

Power series for $\frac{1}{z-2}$, centered at zero

Goal: $\frac{1}{1-z} = 1 + z + z^2 + \ldots$

$r = \frac{1}{2}z$

$$\frac{1}{z-2} = \frac{-1}{2-z} = -\frac{1}{2}\left(\frac{1}{1-\frac{1}{2}z}\right)$$

$$= -\frac{1}{2}\left(1 + \frac{1}{2}z + \frac{1}{4}z^2 + \ldots\right)$$

Power series for $\frac{1}{z-2}$, centered at 1

Goal $\frac{1}{z-2} = \frac{a}{1-b(z-1)}$ $r = b(z-1)$

$= a\left(1 + b(z-1) + b^2(z-1)^2 + \cdots\right.$

$\frac{1}{z-2} = \frac{-1}{2-z}$

$= \frac{-1}{1-z+1}$

$= \frac{-1}{1-(z-1)}$ $r = z-1$

$= -\left(1 + (z-1) + (z-1)^2 + (z-1)^3 + \cdots\right)$

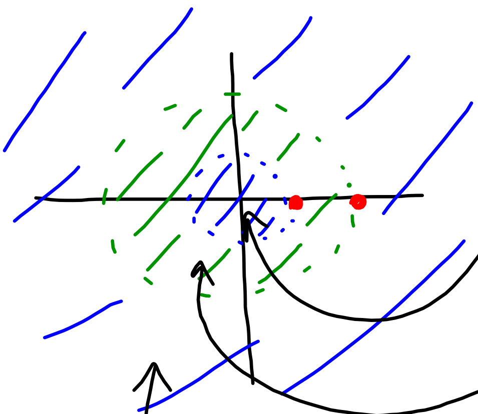
Only good where $|r|<1$

$|z-1|<1$

within 1 unit of 1

Consider again $f(z) = \dfrac{-1}{(z-1)(z-2)}$

$$= \frac{1}{z-1} - \frac{1}{z-2}$$



Power series here, centered at $z_0 = 0$,

Laurent series here, also centered at $z_0 = 0$

Another Laurent series here

$$f(z) = \frac{1}{z-1} - \frac{1}{z-2} \quad \text{in} \quad |z| < 1$$

$$= -\frac{1}{1-z} + \frac{1}{2} \cdot \frac{1}{1-\frac{1}{2}z}$$

$$= -\left(1+z+z^2+z^3+\cdots\right) + \frac{1}{2}\left(1+\frac{1}{2}z+\frac{1}{4}z^2+\cdots\right)$$

$$= -1-z-z^2-z^3-\cdots+\frac{1}{2}+\frac{1}{4}z+\frac{1}{8}z^2+\cdots$$

$$= \left(\frac{1}{2}-1\right)+\left(\frac{1}{4}-1\right)z+\left(\frac{1}{8}-1\right)z^2+\cdots$$

$$= \sum_{k=0}^{\infty}\left(\frac{1}{2^{k+1}}-1\right)z^k$$

$$f(z) = \frac{1}{z-1} - \frac{1}{z-2} \quad \text{in } 1<|z|<2$$

$$= \frac{1}{z}\cdot\frac{1}{1-\frac{1}{z}} + \frac{1}{2}\,\frac{1}{1-\frac{1}{2}z} \qquad \left|\frac{1}{z}\right|<1$$

$$= \frac{1}{z}\left(1+\frac{1}{z}+\frac{1}{z^2}+\frac{1}{z^3}+\cdots\right)$$

$$\left|\tfrac{1}{2}z\right|<1$$

$$\tfrac{1}{2}|z|<1$$

$$|z|<2$$

$$+\frac{1}{2}\left(1+\frac{1}{2}z+\frac{1}{4}z^2+\frac{1}{8}z^3+\cdots\right)$$

$$= \cdots + \frac{1}{z^4}+\frac{1}{z^3}+\frac{1}{z^2}+\frac{1}{z}+\frac{1}{2}+\frac{1}{4}z+\frac{1}{8}z^2+\cdots$$

$k=-2$ $\quad k=-1$ $\quad k=0$ $\quad k=1$ $\quad k=2$

$$c_k = \begin{cases} 1 & \text{for } k=-1,-2,-3,\ldots \\ \frac{1}{2^{k+1}}, & \text{for } k=0,1,2,3,\ldots \end{cases}$$

$$f(z) = \frac{1}{z-1} - \frac{1}{z-2} \qquad |z| > 2$$

$$\left|\frac{1}{z}\right| < \frac{1}{2}$$

$$\frac{1}{z-2} = \frac{1}{z} \cdot \frac{1}{1-\frac{2}{z}} \qquad \left|\frac{1}{z}\right| < \frac{1}{2}$$

$$\left|\frac{2}{z}\right| < 1$$

$$= \frac{1}{z}\left(1 + \frac{2}{z} + \frac{4}{z^2} + \frac{8}{z^3} + \cdots\right)$$

For Monday,

Read Sec. 48 of CB

Try Exercises 3 and 4

Factor z out of the denominator

$$-i^2 \neq (-i)^2$$