

$$f(z) = \frac{1}{1-z}$$

$$f(z) = 1 + z + z^2 + z^3 + \dots \quad |z| < 1$$

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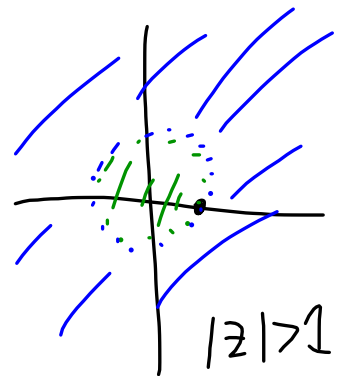
$$= \frac{1}{z} \cdot \frac{1}{1-\frac{1}{z}}$$

$$= -\frac{1}{z} \cdot \frac{1}{1-\frac{1}{z}}$$

$$= -\frac{1}{z} \left( 1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \dots \right)$$

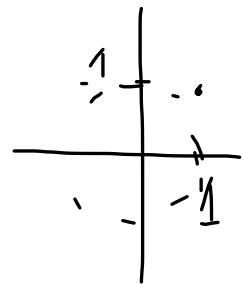
$$= -\frac{1}{z} - \frac{1}{z^2} - \frac{1}{z^3} - \frac{1}{z^4} - \dots$$

$$\frac{1}{1-z} = -\sum_{k=1}^{\infty} \frac{1}{z^k} \quad \text{if } |z| > 1$$



$|z| > 1$

$|z| < 1$



Laurent series for

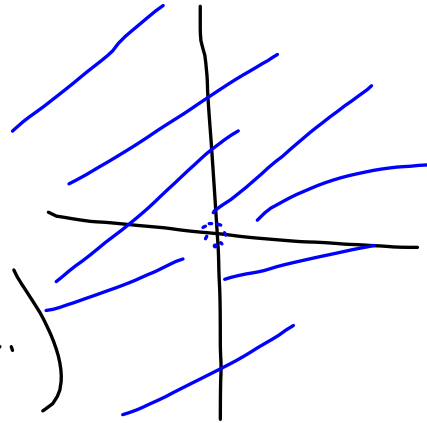
$$\frac{\cos z}{z^2} \quad \text{centered at } z_0 = 0$$

$$\frac{\cos z}{z^2} = \frac{1}{z^2} \cos z$$

$$= \frac{1}{z^2} \left( 1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \frac{z^6}{6!} + \dots \right)$$

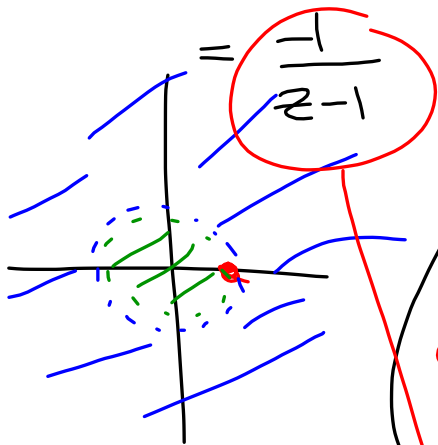
$$= \frac{1}{z^2} - \frac{1}{2!} + \frac{z^2}{4!} - \frac{z^4}{6!} + \dots$$

$$= \sum_{k=-1}^{\infty} \frac{(-1)^{k+1}}{(2k+2)!} z^{2k}$$



$$f(z) = \frac{1}{1-z}$$

Find Laurent series centered at  $z_2 = 1$



$$c_{-1} = -1$$

$$c_k = 0 \text{ if } k \neq -1$$

$$\sum_{k=-\infty}^{\infty} c_k (z-1)^k$$

$$\dots + \frac{c_{-2}}{(z-1)^2} + \frac{c_{-1}}{z-1} + c_0 + c_1(z-1) + c_2(z-1)^2 + \dots$$

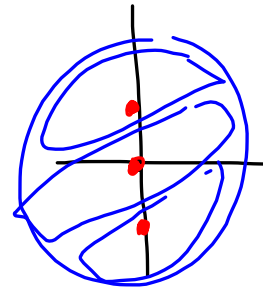
$$g(z) = \frac{1}{z^2}$$

$$\dots + \frac{c_{-2}}{z^2} + \frac{c_{-1}}{z} + c_0 + c_1 z + c_2 z^2 + \dots$$

$$f(z) = \frac{z+1}{z-1}$$

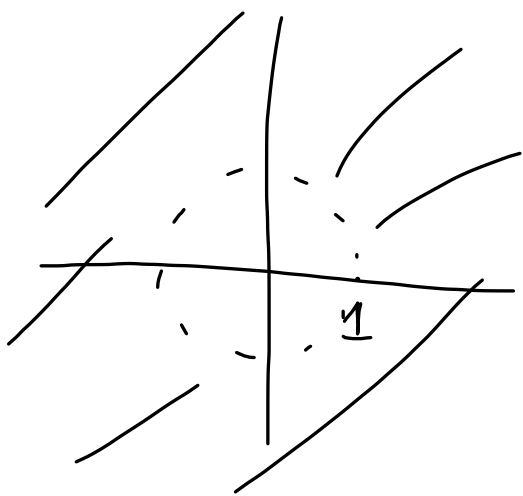
$$\int \frac{dz}{z(1+z^2)}$$

$$= \underbrace{(z+1)} \frac{1}{z-1}$$

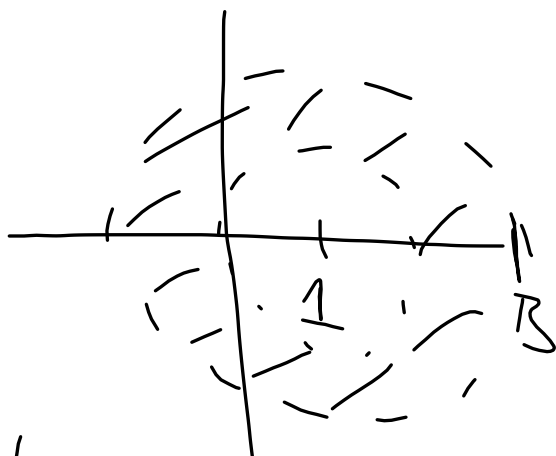


$$+ \frac{c_{-1}}{z-1} + \underbrace{c_0 + c_1(z-1) + c_2(z-1)^2 + \dots}_{\substack{(z-1)+2 \\ 2+(z-1)}} = \left[ 2 + (z-1) \right] \cdot \frac{1}{z-1} = \frac{2}{z-1} + 1$$

$c_{-1} = 2$   
 $c_0 = 1$



$$|z| > 1$$



$$1 < |z-1| < 2$$