Find the zeros of each:

$$f(z)=z^2+2z+1$$
$$=(z+1)(z+1)$$
$$=(z+1)^2$$

$z=-1$ is a zero of order 2

$$g(z)=z^2+4$$
$$=z^2-(-4)$$
$$=(z+2i)(z-2i)$$
$$z=2i,-2i$$

$$h(z)=z^2-6z+13$$
$$=(z-3+2i)(z-3-2i)$$

$$\boxed{x^3+\text{stuff}=0}$$

$$(x\pm\ )(x^2+\text{stuff})$$

Galois

Group Theory

$f(z) = z^2 + 2z + 1$ zero at $z = -1$

$= (z+1)(z+1)$

$f(z) = (z+1)^m g(z)$

$f(-1) = 0$

$f'(z) = 2z + 2$

$f'(-1) = 0$

$f''(z) = 2 \neq 0$ for any z

$z = -1$ is a zero of order 2

$f(z) = (z+1)^2 \cdot 1$

$z_0 = -1$ $g(z) = 1$

$f(z) = z^2+4 = (z+2i)(z-2i)$

zero of $-2i = z_0$

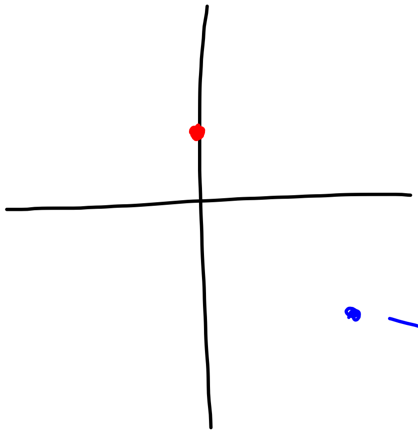
Definition: $f(-2i) = 0$ → $-2i$ is a zero of order 1

$f'(z) = 2z$

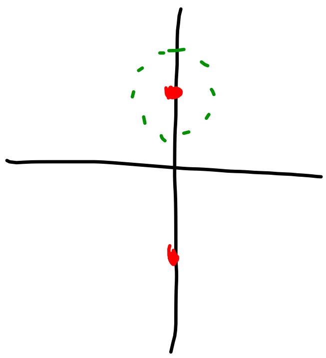
$f'(-2i) = -4i \neq 0$ → of order 1.

Thm A.2: $f(z) = (z+2i)^{1}\underbrace{(z-2i)}_{g(z)}$ → $-2i$ is a zero of order 1

$g(-2i) \neq 0$



$$g(z)=\frac{1}{z^2+4}=\frac{1}{(z+2i)(z-2i)}$$



z_0 , $f(z_0)=0$

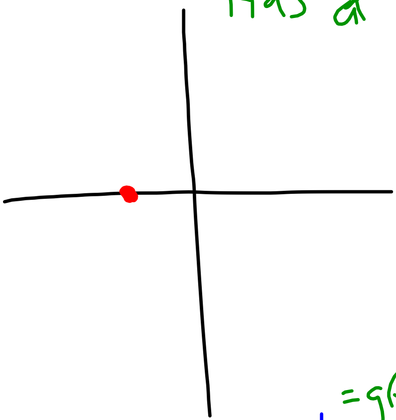
$$f(z)=\frac{1}{z-i}$$

has an isolated singularity at $z=i$

$$f(z) = \frac{1}{z^2+2z+1} = \frac{1}{(z+1)(z+1)}$$

$z=-1$ is a "bad spot" (singularity)

Has a Laurent series centered at $z=-1$ pole of order 2



$$\sum_{k=-\infty}^{\infty} c_k (z+1)^k$$

$$f(z) = \frac{1 \;\; {}^{=g(z)}}{(z+1)^2}$$

$c_{-2} = 1$, $c_k = 0$ for all other values of K.

$$f(z) = \frac{\sin z}{z^3} = \frac{g(z)}{(z-0)^3} \qquad g(0) = 0$$

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}$$

$z=0$ is a singularity

$$f(z) = \frac{1}{z^3}\left(z - \frac{z^3}{3!} + \frac{z^5}{5!} - \frac{z^7}{7!} + \cdots\right)$$

$$= \frac{1}{z^2} - \frac{1}{3!} + \frac{z^2}{5!} - \cdots$$

$z=0$ is a pole of order 2