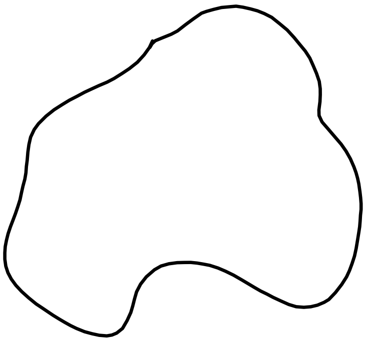
Dirichlet Problem (DP)

real-valued

Region $R \subseteq \mathbb{C}$, boundary function $f(z)$ on ∂R (boundary of R)

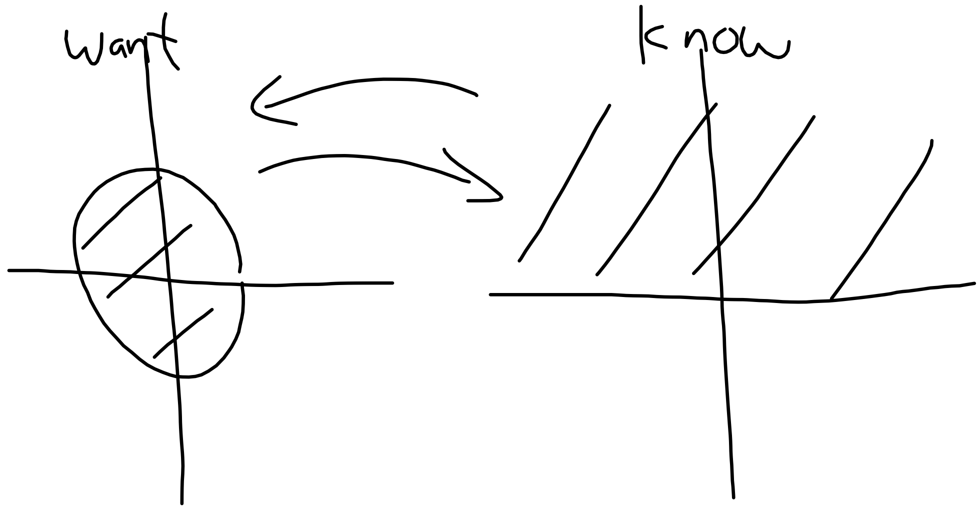


Want real-valued $u(x,y)$ that is

a) harmonic in R and

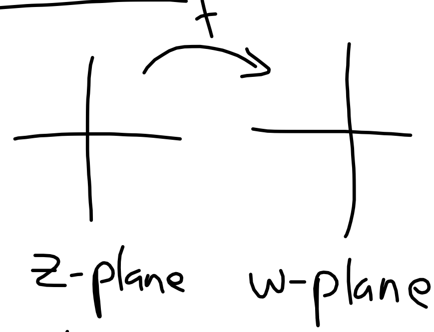
b) "matches" $f(z)$ on ∂R

Idea: We want a solution to DP on some region, know a solution on another region.



Ch. 7 of Churchill + Brown

Goal: $w = \dfrac{Az+B}{Cz+D}$



A, B, C, D are complex constants

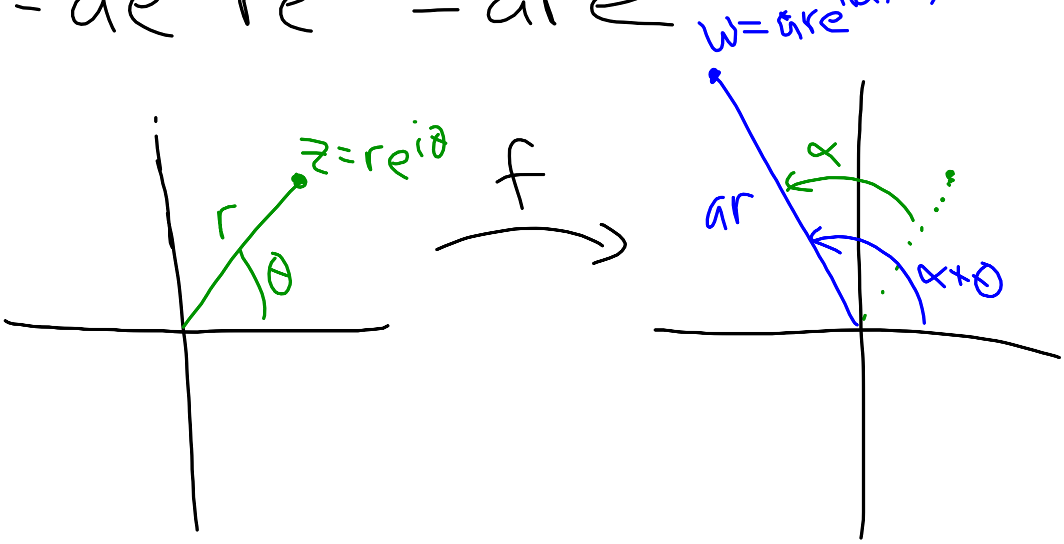
linear fractional transformation

Möbins transformations

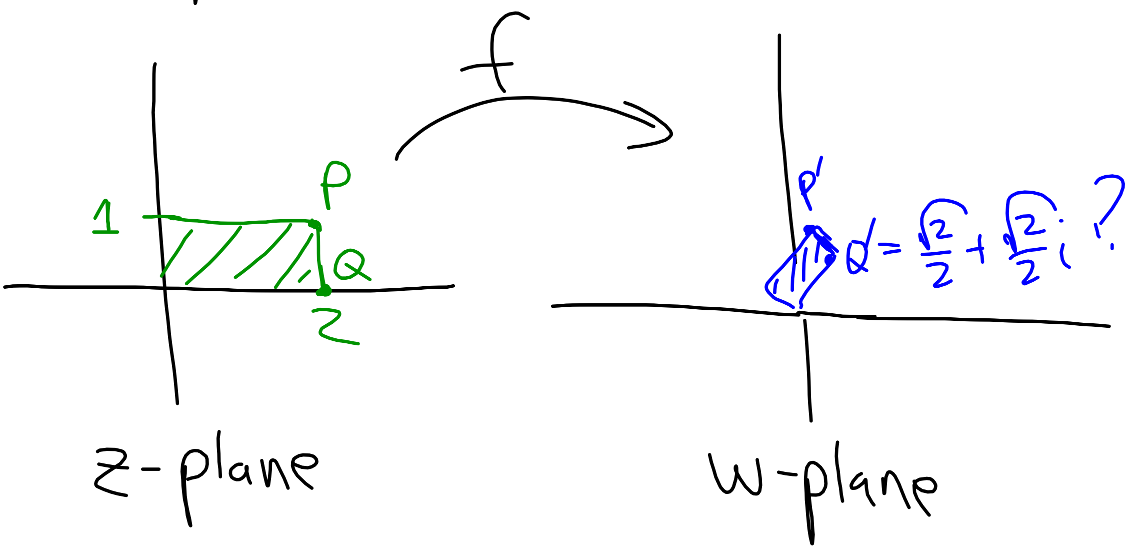
$w = f(z) = Az$ A is complex

Let $A = ae^{i\alpha}$ and $z = re^{i\theta}$

$w = ae^{i\alpha} re^{i\theta} = are^{i(\alpha+\theta)}$



Example: $w = \frac{1}{2} e^{\frac{\pi}{4} i} z = f(z)$



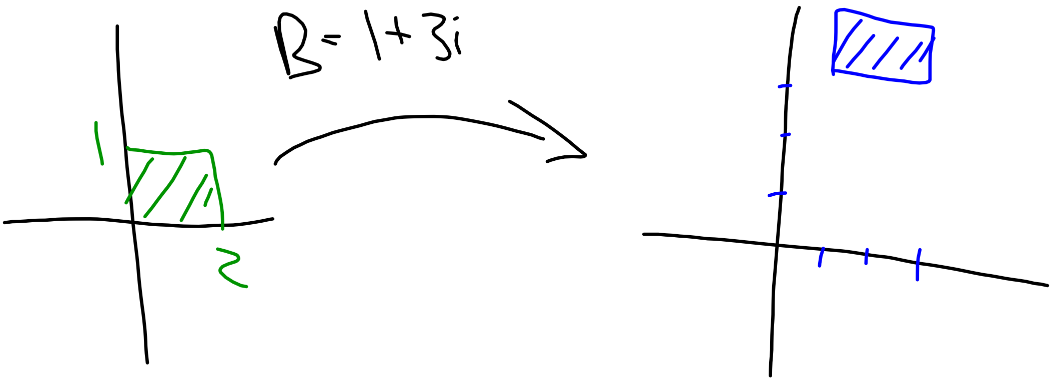
$$f(2+0i) = \left(\frac{1}{2} e^{\frac{\pi}{4} i}\right)(2) = e^{\frac{\pi}{4} i} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i$$

$w = z + \beta$, β is complex

$\beta = b + \beta i$

$w = x + iy + b + \beta i$

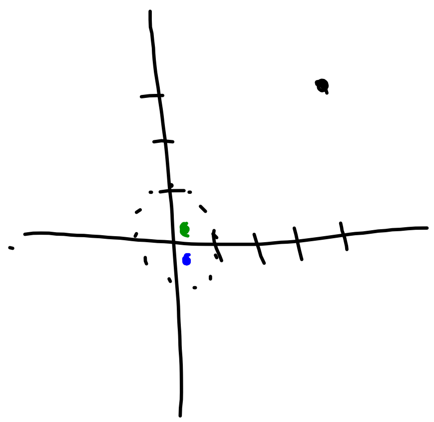
$w = x + b + i(y + \beta)$ translation



$w = Az + B$ A first, then B

$$W = \frac{1}{z} = \frac{\bar{z}}{z\bar{z}} = \frac{\bar{z}}{|z|^2} = \frac{\bar{z}}{|z|^2} = \overline{\left(\frac{z}{|z|^2}\right)}$$

$$z = 4 + 3i \quad , \quad |z|^2 = 25 \quad , \quad \frac{z}{|z|^2} = \frac{4}{25} + \frac{3}{25}i$$



$f(z) = \frac{1}{z}$

$g(z) = z$