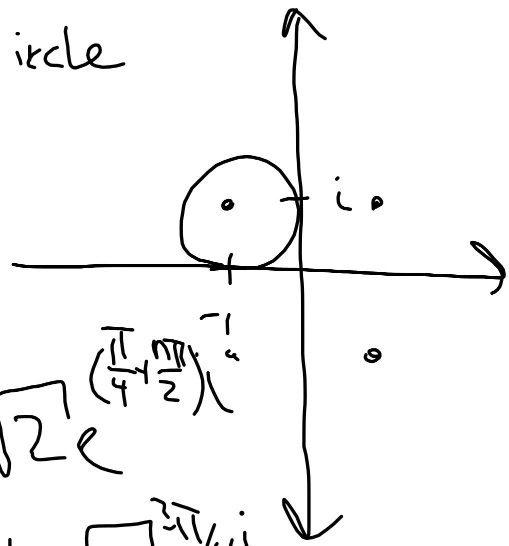
$\int \frac{dz}{z^4+4}$ $\quad\gamma$ is the circle

$z_0 = \sqrt{2}e^{\frac{3\pi}{4}i}$



$= 2\pi i \cdot \mathrm{Res}(f; z_0)$

$= 2\pi i \cdot \frac{1}{4(\sqrt{2}e^{\frac{3\pi}{4}i})^3}$

$= \frac{\sqrt{2}\pi i}{8e^{\frac{\pi}{4}i}}$

$(-4)^{1/4} = \sqrt{2}e^{(\frac{\pi}{4}+\frac{n\pi}{2})i}$

$n=1: \sqrt{2}e^{3\pi/4 i}$

Thrm B2 $\quad g(z)=1$

$h(z)=z^4+4$

$$\int_\gamma \frac{1}{(z+(1+i))(z-(1+i))(z+(1-i))(z-(1-i))}$$

$$\int_\gamma \frac{\left(\frac{1}{(z+(1+i))(z-(1+i))(z-(1-i))}\right)}{z+(1-i)}$$

$$2\pi i \frac{1}{((-1+i)+(1+i))((-1+i)-(1+i))((-1+i)-(1-i))}$$

$$2\pi i\left(\frac{1}{2i(-2)(-2+2i)}\right)=\frac{\pi}{-4i+4}$$ Rationalize. . . . .

Zahlen $\frac{\sqrt{2}\pi i}{8e^{\frac{9\pi}{4}i}}$ $\sqrt[4]{4} = \sqrt{2}$

Nick $\frac{\pi}{-4i+4}$ $z^4+4 = (z \quad)(z \quad)(z \quad)(z \quad)$

Miriam $\frac{\pi i}{4i+4}$

Gregg $4^{-\frac{7}{4}}\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)$

$$\operatorname{Res}(f; z_0) = \frac{g(z_0)}{h'(z_0)} \qquad z^3 + z = z(z+i)(z-i)$$

$$\operatorname{Res}\left(\frac{e^z}{z^3+z}; 0\right)$$

$$\frac{e^z}{z^3+z} = \frac{e^z/(z^2+1)}{z} \Longrightarrow \text{Residue is } \left.\frac{\frac{e^z}{z^2+1}}{1}\right|_{z=0} = 1$$

$$\text{Residue is } \left.\frac{e^z}{3z^2+1}\right|_{z=0} = \frac{1}{1} = 1.$$

$$\operatorname{Res}\left(\frac{e^z}{z^3+z}\right) = \lim_{z\to 0} z \cdot \frac{e^z}{z^3+z} = \lim_{z\to 0} \frac{e^z}{z^2+1} = 1$$

$$\lim_{z\to -1} \frac{z^3+1}{z^2(z+1)} = \frac{(z+1)(z^2-z+1)}{z^2(z+1)} = \lim_{z\to -1} \frac{z^2-z+1}{z^2}$$

$\lim_{z\to -1}$

$$= 3$$

Assignment 25 (last one!)

4G: pg. 114: 13(b), (d)

pg. 115 14(b), (d)

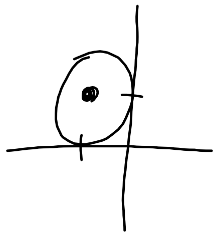
13 d residue at $z=0$ for

$$e^{1-\frac{1}{z}} = e^{1}\underbrace{e^{-\frac{1}{z}}}$$

$$e^{z}$$

$$e^{-\frac{1}{z}} = 1 + \left(-\frac{1}{z}\right) + \frac{1}{2}\left(-\frac{1}{z}\right)^2$$

14a) $\displaystyle\int_\gamma \frac{1}{z^4+4}\,dz$



$$\frac{1}{z^4+4} = \frac{1}{(z^2+2i)(z^2-2i)}$$

$$= \frac{1}{(z-(1-i))(z+(1-i))(z-(1+i))(z+(1+i))}$$

$$\int_\gamma \frac{1}{z^2+4}\,dz = 2\pi i\,\mathrm{Res}(f;-1+i)$$

$$\mathrm{Res}(f;-1+i) = \left.\frac{1/(z-1+i)(z-1-i)(z+1+i)}{1}\right|_{z=-1+i}$$

$$= \frac{1}{(-1+i-1+i)(-1+i-1-i)(-1+i+1+i)}$$

$$= \frac{1}{(-2+2i)(-2)(2i)}$$

$$= \frac{1}{-4i(-2+2i)}$$

$$= \frac{1}{8i+8}$$

$$\int_\gamma \frac{1}{z^4+4}\,dz = 2\pi i\left(\frac{1}{8i+8}\right)$$

$$= \frac{\pi i}{4+4i}\,\frac{(4-4i)}{(4-4i)}$$

$$= \frac{4\pi i+4\pi}{32}$$

$$= \frac{\pi i+\pi}{8}$$