$u(x,y) = x^2 - y^2$ Find the harmonic conjugate

$$\frac{\partial u}{\partial x} = 2x = \frac{\partial v}{\partial y} \Longrightarrow v = 2xy + \phi(x)$$

$$\frac{\partial v}{\partial x} = 2y + \phi'(x) = -\frac{\partial u}{\partial y} = -(-2y) + 0$$

$$= 2y + 0$$

$$\phi'(x) = 0 \Rightarrow \phi(x) = C$$

$$v(x,y) = 2xy + C$$

$$f(z) = z^2 = (x+iy)^2$$

$$= (x+iy)(x+iy) = \underbrace{x^2-y^2}_{u} + \underbrace{2xy}_{v}\,i$$

$u(x,y) = x^2 - y^2$ electric field

Level curves: $x^2 - y^2 = c$

$c = 0$: $x^2 = y^2 \implies x = \pm y$

$c = 1$: $x^2 - y^2 = 1$

$(1,0)$ $(-1,0)$

$x^2 = y^2 + 1$

$v(x,y) = 2xy$

$c = 0$: $2xy = 0$

$c = 2$: $2xy = 2$

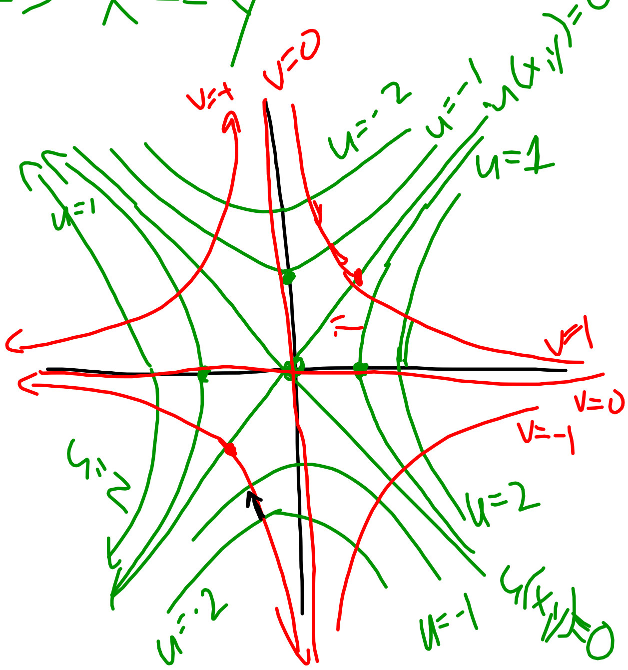
$xy = 1$

$y = \frac{1}{x}$

$c = -2$: $2xy = -2$

$xy = -1$

$y = -\frac{1}{x}$



$$\int_\gamma \frac{1}{z^2 \sin z}\, dz \qquad \gamma \text{ is } |z| = 1$$

$\to$ Pole of order 3 at $z=0$.

$$z^2 \sin z = z^2\left(z - \frac{z^3}{3!} + \frac{z^5}{5!} - \cdots\right)$$

$$= (z-0)^3 \cdot \underbrace{(1 - \text{stuff})}_{\text{analytic, not zero @ } z=0}$$

Has a zero of order 3 @ $z=0$

$$\frac{1}{z^2 \sin z} = \frac{1}{z^2} \cdot \frac{1}{\sin z} = \frac{1}{z^2}\left(\frac{1}{z} + \frac{z}{3!} +\right.$$

$$z - \frac{z^3}{3!} + \frac{z^5}{5!} - \cdots$$

$$\begin{array}{l} \frac{1}{z} + \frac{z}{3!} + \text{stuff} \\ \hline \big) \; 1 \\ \;\; 1 - \frac{z^2}{3!} + \frac{z^4}{5!} - \cdots \\ \hline \;\; \frac{z^2}{3!} - \frac{z^4}{5!} + \end{array}$$

Residue

$$\frac{1}{1^2 \sin(1)} = 1.188$$

$$\frac{1}{1} + \frac{1}{3!} + \frac{1}{5!} + \frac{1}{7!}$$

$$= \frac{1}{z^3} + \frac{1}{3!} \cdot \frac{1}{z} + \frac{1}{5!} z$$

?