$$\int_{|z|=1} \frac{dz}{z^2 \sin z} = 2\pi i \cdot \frac{1}{6} = \frac{\pi}{3} i \approx 1.047197\ldots i$$

$$\gamma(\theta) = e^{i\theta}, \quad 0 \le \theta \le 2\pi$$

$$\frac{1}{z^2 \sin z} = \frac{1}{z^3} + \frac{1}{3!} \cdot \frac{1}{z} + \frac{\;}{(3!)^2 - 5!} z + \frac{\;}{7! - 3!5!} z^3$$

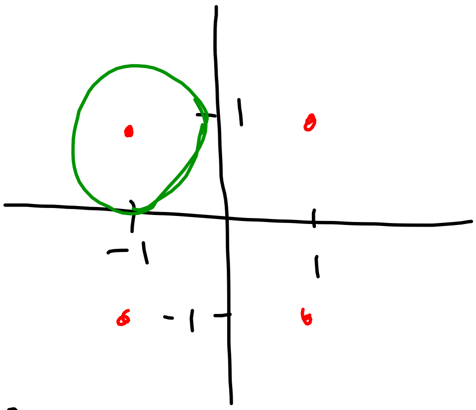
Residue

$$\int_{|z|=1} \frac{dz}{z^2 \sin z} = \int_0^{2\pi} \frac{i e^{i\theta} d\theta}{(e^{i\theta})^2 \sin e^{i\theta}} = i \int_0^{2\pi} \frac{1}{e^{i\theta} \sin e^{i\theta}} d\theta$$

$$\int_\gamma \frac{dz}{z^4+4} = \frac{\pi}{8}(1+i) \quad \text{By Residue Thm.}$$

$\gamma: \ |z+1-i| = 1$

$|z-(-1+i)| = 1$



$\gamma(\theta) = -1 + i + e^{i\theta} \qquad 0 \leq \theta \leq 2\pi$

$$\int_0^{2\pi} \frac{ie^{i\theta}\,d\theta}{(-1+i+e^{i\theta})^4+4} = \frac{\pi}{8}(1+i)$$

$$\int_{C_r(z_0)} \frac{f(z)}{z-z_0}\,dz = 2\pi i\, f(z_0)$$

$$\int_{C_1(-1+i)} \frac{1}{z^4+4}\,dz = \int_{C_1(-1+i)} \frac{1/[(z-(1+i))(z-(1-i))(z-(-1-i))]}{z-(-1+i)}\,dz$$

$$= 2\pi i \cdot \frac{1}{((-1+i)-(1+i))((-1+i)-(1-i))((-1+i)-(-1-i))}$$

$$= \frac{\cancel{2}\pi \cancel{i}}{(-2)(-2+2i)(\cancel{2i})}$$

$$= \frac{\pi}{4-4i}$$

$$= \frac{\pi}{(4-4i)} \cdot \frac{(4+4i)}{4+4i}$$

$$= \frac{\pi(4+4i)}{32} = \frac{\pi}{8}(1+i)$$

Final 2-4 Wed

Owens, across from 122

* Since Exam 2 50%?
  * harmonic functions: is it harmonic? harmonic conjugate.
* Residues: finding, justifying
* Identifying & justifying singularities

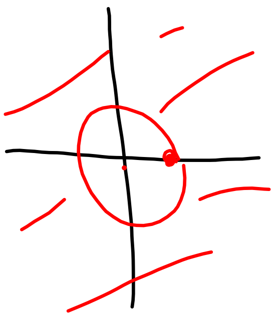
Before that:

* complex # stuff
* holomorphic functions — CR-eqns, give derivative check
* functions: e^z, $\sin z$, $\cos z$ (limited)
* contour integration
  * "bare-handed"
  * using CIFs
* Power & Laurent series

$\cancel{\frac{1}{z}}$ $\quad \frac{1}{1-z} = 1 + z + z^2 +$

office
Mon 10-12, 2-?
Tues 10-2
Wed 10-2

$$\frac{1}{z(z-1)} = \frac{1}{z} \cdot \frac{1}{z-1} = \frac{1}{z^2} \frac{1}{1-\frac{1}{z}} \qquad r = \frac{1}{z}$$



$$= \frac{1}{z^2}\left(1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots\right) \qquad |r| = \left|\frac{1}{z}\right| = \frac{1}{|z|} < 1$$

$$= \frac{1}{z^2} + \frac{1}{z^3} + \cdots \qquad \text{if } |z| > 1$$