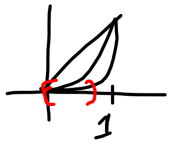
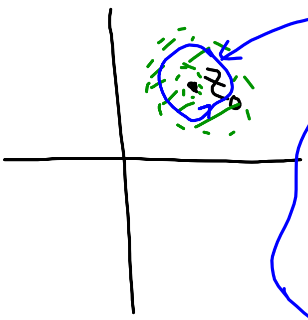
$$\int_{C_r(z_0)} (z-z_0)^n \, dz = \begin{cases} 0 & \text{if } n \neq -1 \\ 2\pi i & \text{if } n = -1 \end{cases}$$



f has an isolated singularity at z_0.



Laurent series here.

Region $\{z : R_1 \le |z - z_0| \le R_2\}$

f converges uniformly in the region.

$$f(z) = \sum_{k=-\infty}^{\infty} c_k (z-z_0)^k$$

$$\int_\gamma f(z)\,dz = \int_\gamma \left[\sum_{k=-\infty}^{\infty} c_k (z-z_0)^k \right] dz$$

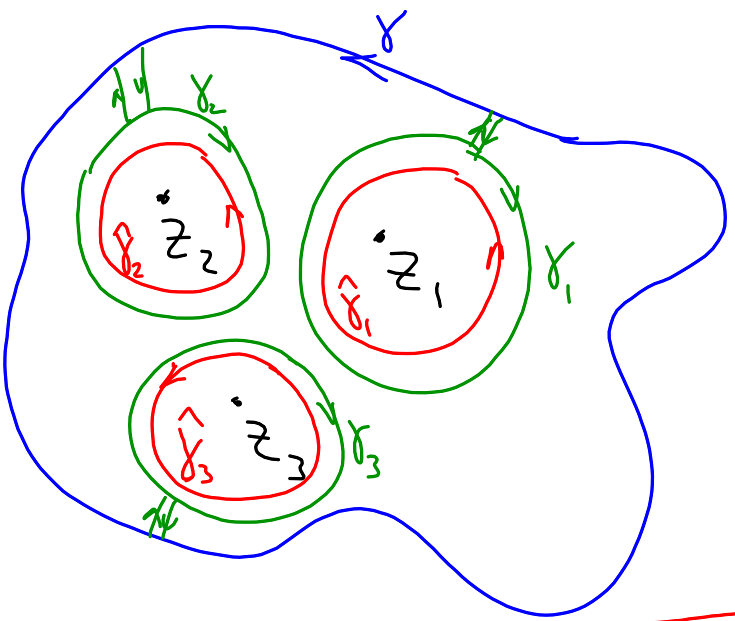
$$= \sum_{k=-\infty}^{\infty} \left[\int_\gamma c_k (z-z_0)^k \, dz \right]$$

$$= \underline{\underline{c_{-1}}} \cdot 2\pi i$$

Residue of f at z_0 singularity

Notation: $\mathrm{Res}(f; z_0)$

f has singularities @ z_1, z_2, z_3



$$\int_\gamma f(z)\,dz = ?$$

BG 

$$\int_\gamma f(z)\,dz = \int_\gamma f + \overbrace{\int_{\gamma_1} f + \int_{\gamma_2} f + \int_{\gamma_3} f}^{\text{sum}=0} + \int_{\hat\gamma_1} f + \int_{\hat\gamma_2} f + \int_{\hat\gamma_3} f$$

Residue Theorem

$$= \cancel{\int_{BG} f}^{\,\to 0} + \int_{\hat\gamma_1} f + \int_{\hat\gamma_2} f + \int_{\hat\gamma_3} f$$

$$= 2\pi i \,\mathrm{Res}(f; z_1) + 2\pi i \,\mathrm{Res}(f; z_2) + \triangle$$

$$= 2\pi i \sum_{i=1}^{n} \mathrm{Res}(f; z_i)$$

Finding residues:

① Find Laurent series, residue is c_{-1}

② Lemma 9.12 (pg. 109 of 4G)

f & g are holomorphic at z_0, $f(z_0) \neq 0$
and z_0 is a zero of order 1 of g at z_0.
z_0 is a pole of order 1 of $\frac{f}{g}$

$$\text{Res}\left(\frac{f}{g}; z_0\right) = \frac{f(z_0)}{g'(z_0)}$$

Example: $f(z) = \frac{z}{z^2+1} = \frac{z}{(z+i)(z-i)}$

At $z_0 = -i$, $f(z) = \frac{z/(z-i)}{z+i}$

$$\text{Res}(f; -i) = \frac{-i/(-i-i)}{1} = \frac{-i}{-2i} = \frac{1}{2}$$

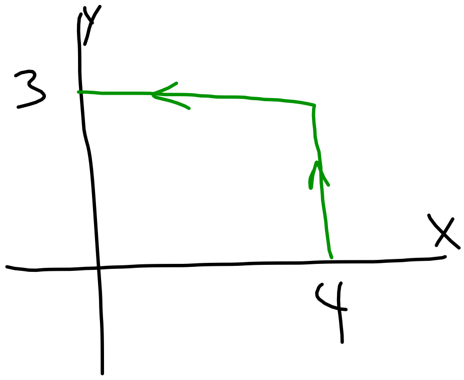
Turn in Wed 3/5 by 3:00

From the online 4G book:

pgs. 114+115: 13(a),(c)
14(a),(c)

All with work/justification.

Exam 2: #13



Want eventually:
bound for

x+iy

$$\int_\gamma z^2 e^z \, dz$$

$$|e^z| = |e^{x+iy}|$$