Laplace transform

$$f(t) \xmapsto{\text{Laplace transform}} F(s)$$

$$\xleftarrow[\text{inverse Laplace transform}]{}$$

$F(s) \to \boxed{\mathcal{L}^{-1}} \to f(t)$

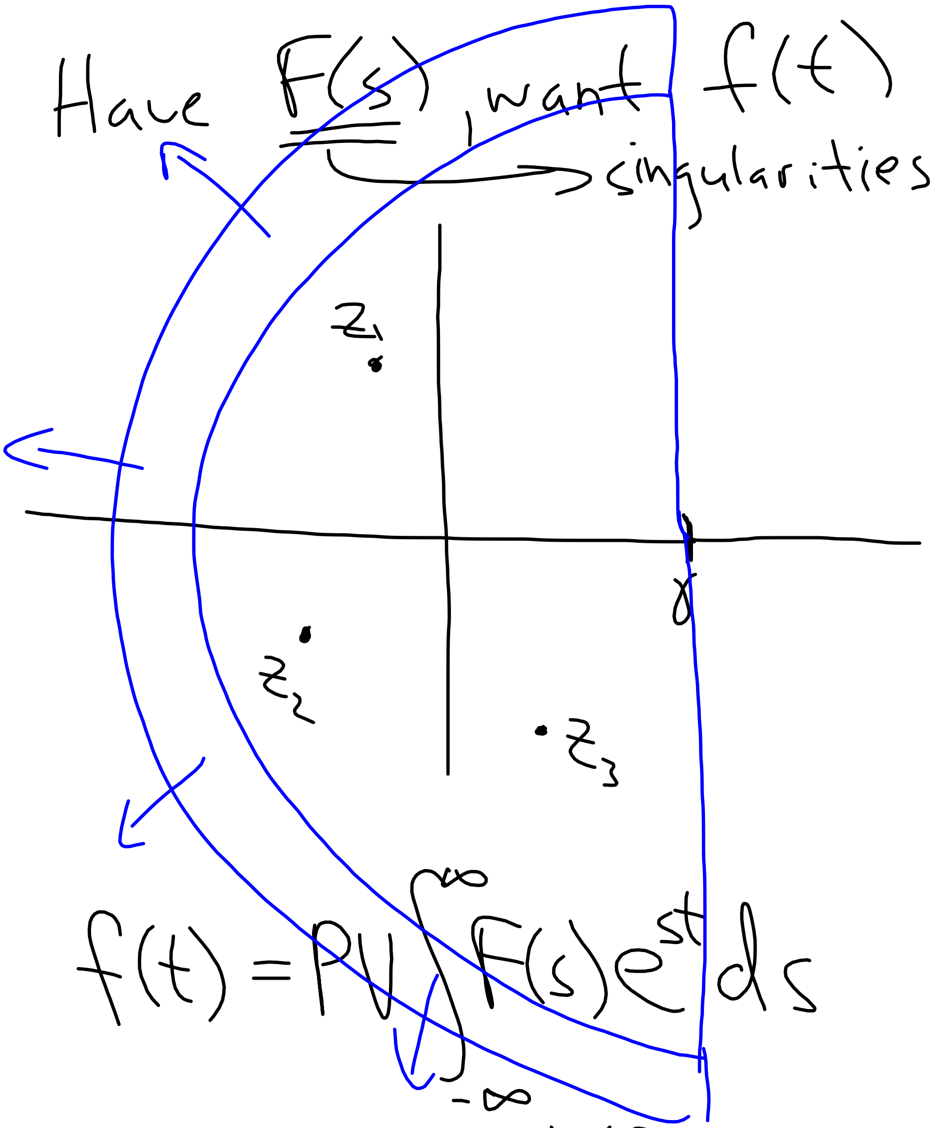
$f(t) \to \boxed{\mathcal{L}} \to F(s)$

$f(t)$	$F(s)$

$$F(s) = \int_0^\infty f(t)e^{-st}\,dt \qquad s > 0$$

Have $F(s)$, want $f(t)$

singularities at z_1, z_2, z_3



$$f(t) = PV\int_{-\infty}^{\infty} F(s)e^{st}\,ds$$

$$= \lim_{R\to\infty} \int_{\gamma-iR}^{\gamma+iR} F(s)e^{st}\,ds$$

$$2\pi i \sum \underbrace{\text{Res}}_{\text{of } F(s)e^{st}} = \int$$

$$= \underbrace{\int}_{\to \text{ inverse LT as } R\to\infty} + \underbrace{\int}_{\to 0 \text{ as } R\to\infty}$$

$$a^2-b^2=(a+b)(a-b)$$
$$a^2+b^2=$$
$$a^3+b^3=(a+b)(a^2-ab+b^2)$$
$$a^3-b^3=(a-b)(a^2+ab+b^2)$$

$$F(s)=\frac{12}{s^3+8}$$

$$=\frac{12}{(s+2)(s^2-2s+4)}$$

$$=\frac{A}{s+2}+\frac{Bs+C}{s^2-2s+4}$$

$$=\frac{1}{s+2}+\frac{4-s}{s^2-2s+4}$$

$$=\frac{1}{s-(-2)}+\frac{4-s}{s^2-2s+1+3}$$

$$4-s \longrightarrow -(s-4)$$
$$-(s-1-3)$$
$$-(s-1)+3$$

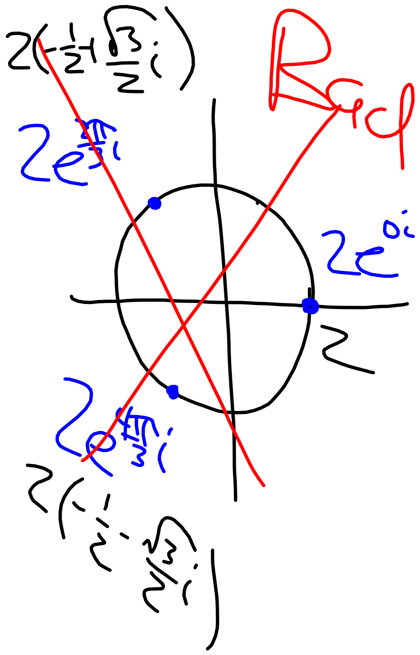
$$=\frac{1}{s-(-2)}+\frac{-(s-1)+3}{(s-1)^2+(\sqrt{3})^2}$$

$$=\frac{1}{s-(-2)}-\frac{(s-1)}{(s-1)^2+(\sqrt{3})^2}+\sqrt{3}\,\frac{\sqrt{3}}{(s-1)^2+(\sqrt{3})^2}$$

$$\boxed{f(t)=e^{-2t}-e^{t}\cos\sqrt{3}t+\sqrt{3}e^{t}\sin\sqrt{3}t}$$

$$F(s) = \frac{12}{s^3+8} \qquad (e^{i\theta})^2 = e^{2\theta i}$$

$$\mathcal{L}^{-1}[F(s)] = PV\int_{-\infty}^{\infty} \frac{12}{s^3+8} e^{st}\, ds$$



$$= 2\pi i \sum_{k=1}^{n} \operatorname{Res}\left(\frac{12e^{st}}{s^3+8}; s_k\right)$$

$$(s+2)(s+1+\sqrt{3}i)(s+1-\sqrt{3}i)$$

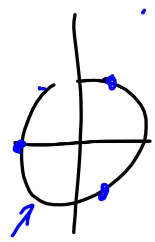
$$\operatorname{Res}\left(\frac{f}{g}(z_0)\right) = \frac{f(z_0)}{g'(z_0)} \qquad g'(z) = 3z^2$$

$$f(z) = 12e^{zt} \qquad g(z) = z^3+8 \qquad g'(z) = 3z^2$$

Poles: $z = -2,\ 1+\sqrt{3}i,\ 1-\sqrt{3}i$

$$\text{Res}\left(\frac{f}{g};2\right) = \frac{12e^{-2t}}{3(-2)^2} = e^{-2t}$$

$$\text{Res}\left(\frac{f}{g}:1+\sqrt{3}i\right) = \frac{\overset{4}{\cancel{12}}e^{(1+\sqrt{3}i)t}}{\cancel{3}(1+\sqrt{3}i)^2} =$$



$$= \frac{12e^{t}e^{\sqrt{3}it}}{3(2e^{\frac{\pi}{3}i})^2}$$

$$= e^{-\frac{2\pi}{3}i}e^{t}e^{\sqrt{3}it}$$

$$= e^{t}\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)e^{\sqrt{3}it}$$

$$\text{Res}\left(\frac{f}{g};1-\sqrt{3}i\right) = e^{t}\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)e^{\sqrt{3}it}$$

$$-e^{t}e^{\sqrt{3}it}$$

$$-e^{t}\left(\cos\sqrt{3}t+i\sin\sqrt{3}t\right)$$