Math 451 Assignment 12, Spring 2014 **Parts due at various times**

In this assignment you will be writing a function file that performs back substitution on the matrix obtained by row-reducing a matrix to obtain the solution to a system of equations. You'll start by doing this for a system of three equations in three unknowns, then create a more general function that can be given an $n \times n + 1$ augmented matrix and return the solution to the system of equations corresponding to the augmented matrix.

1. Suppose that a system of three equations in the three unknowns x_1, x_2 and x_3 row-reduces to

$$\begin{bmatrix} 1 & 6 & -2 & 7 \\ 0 & 8 & 1 & -5 \\ 0 & 0 & -2 & 8 \end{bmatrix}$$

   Do the following as a pencil-and-paper exercise and turn it in at the start of class on Wednesday, May 7th. You should create a second copy for you to keep and refer to while doing the other parts of this assignment.

   (a) Write an equation for x_3 in terms of the entries in the matrix.

   (b) Write an equation for x_2 in terms of the entries in the matrix and x_3 (as a symbol, not its actual value).

   (c) Write an equation for x_1 in terms of the entries in the matrix, x_2 and x_3 (again as symbols).

   (d) Give the solution to the system as a **column** vector $\mathbf{x}$ whose components are the numerical values of x_1, x_2 and x_3.

2. Write a script file named *your firstname_bs1* (that's BS for "back-substitution!") that does everything you did for Exercise 1, for the augmented matrix given. You might wish to begin with numerical values for the entries in the augmented matrix, but then you should *change those numbers to references to entries in the matrix.* The output should be the 3×1 solution vector, which you should initialize as zeros in your file.

   Once you have that working correctly, change your script to a function file that accepts a 3×4 augmented matrix in upper triangular form and returns its solution. E-mail this to me **by 5 PM on Thursday, May 8th**.

3. We wish to create a function file that will do this for any $n \times (n+1)$ upper triangular matrix - this can be a little tricky. If you wish you can try it on your own, or you can just skip ahead to the next part. If you try it yourself, you need to create two nested loops to perform the back substitution. I'd try to make it work for the 3×4 case, then adapt that to work for any $n \times (n+1)$ matrix.

4. I have created a function file *gw_bs2.m* that does what your file from Exercise 2 does, but using loops. Download it from the 5/5 date on the class schedule and rename it *yourfirstname_bs2.* Figure out how to modify it to accept any $n \times (n+1)$ upper triangular matrix and return the solution to the system. When it works, save it and send it to me **by 4PM on Friday, May 9th**.

5. You now have a function that performs row reduction, and another that back-substitutes. Write a new function *yourfirstname_solve* that accepts as its arguments the coefficient matrix A and right-hand -side vector $\mathbf{b}$ and returns the solution to the system. Send it to me **by noon Sunday, April 11th**.