Math 451 ASSIGNMENT 15, SPRING 2014 **Due at 4 PM Monday, May 19th**

The eventual goal of this assignment is to determine the polynomial of least degree that passes though a given set of points in the xy-plane. Such a polynomial is called a **Lagrange polynomial**. Look them up in *Wikipedia* and see if you can understand anything written there abut them. In this assignment we'll make sense of all the messy stuff you see there.

NOTE: We will be working throughout with polynomial functions in factored form. Leave then in that form - *DO NOT* multiply them out.

1. Go to the web page *www.desmos.com* and click *Launch Calculator*.

   (a) Enter the function $f(x) = (x+3)(x-1)(x-2)$ and adjust the window (the little wrench icon in the upper right) so that you get a good view of the function. **Draw:** Sketch what you see - don't put a scale on either axis, but label the x-intercepts with their values.

   (b) **Write:** Determine $f(-1)$ from the equation given, showing the result *and exactly how it is obtained*. Does the graph support your result?

   (c) Now Suppose that we wanted to alter our function so that the x-intercepts remain the same, but so that $f(-1) = 1$. This can easily be done: Just divide the current form of the function by $f(-1)$. Add a second function in the *Desmos* window that is $y = f(x)/f(-1)$. Adjust the window as needed to see that it has the same x-intercepts as f, but that it has value 1 at $x = -1$.

   (d) **Write:** Using what you have just discovered/seen, give a polynomial of the form

   $$P(x) = \frac{(x-a_1)(x-a_2)(x-a_3)\cdots}{(x_0-a_1)(x_0-a_2)(x_0-a_3)\cdots} \tag{1}$$

   that has x-intercepts at 1, 2, and 4, and for which $P(3) = 1$. Note that x is the variable, a_1, a_2, a_3, ... are the roots, and x_0 is the x value where we want the polynomial to have value one. Plot your answer with *Desmos* and alter it as needed if it doesn't do what it is supposed to. **Draw:** Sketch what you see.

   (e) Keep $P(x)$ but get rid of the other functions by clicking the $\times$ to the right of each - we don't need them any more. Add a subscript to get $P_3(x)$ by putting the cursor where you want the subscript, holding down the *shift* key and pressing the minus/underscore key.

2. (a) Create a polynomials P_1 that has x-intercepts at 2, 3, 4 and so that $P_1(1) = 1$. Plot it to make sure you succeeded. **Draw:** Sketch the graph.

   (b) Create polynomials P_j, $j = 2, 4$ for which $P_j(j) = 1$ and x-intercepts at all of $x = 1, 2, 3, 4$ except j. **Draw:** Plot all of them and sketch the graph of each.

   (c) **Write:** Consider $y = 3P_2(x)$. What value should y have at each of $x = 1, 2, 3, 4$? Give your answers using function notation.

3. (a) **Write:** What is the degree of each of your polynomials P_j, $j = 1, 2, 3, 4$? If you added them all together, what degree would the degree of the resulting polynomial be?

   (b) Suppose that we let

   $$f(x) = 4P_1(x) + 3P_2(x) + 3P_3(x) + 5P_4(x). \tag{2}$$

   Write: What should $f(x)$ be for $x = 1, 2, 3, 4$? Give your answers using correct function notation, of course.

   (c) **Draw:** Plot f with *Desmos* - you can simply enter (2) and it will use the previously entered $P_j(x)$ functions to create the graph. Sketch what you see.

4. **Write:** Give a polynomial $g(x)$ that is a "blend" of the forms (2) and (1) that passes through $(1, 5)$, $(2, 3)$ and $(3, 7)$. Check it with *Desmos*.

5. Go back to the *Wikipedia* page on Lagrange polynomials and see if it make any more sense now. The third line of mathematics there is pretty much our (1); the symbol $\prod$ is like $\sum$, but means *product* rather than *sum*. The second line of mathematics is our (2).