Math 451 ASSIGNMENT 5 **Due at 4 PM Monday, April 7th**

Do Exercises 1 and 2 on additional paper. Exercise 3 is on the back, and should be done on this paper.

1. For each of the following, sketch the graph (a separate sketch for each situation) of a function on the interval $[a, b]$ meeting the given conditions. If one is not possible, just say " not possible."

   (a) f is continuous on $[a, b]$, $f(a)$ and $f(b)$ are both negative, and f has no root in $[a, b]$.

   (b) f is continuous on $[a, b]$, $f(a)$ is positive and $f(b)$ is negative, f has no root in $[a, b]$.

   (c) $f(a)$ is positive and $f(b)$ is negative, f has no root in $[a, b]$.

   (d) f is continuous on $[a, b]$, $f(a)$ and $f(b)$ are both positive, and f has exactly one root in $[a, b]$.

   (e) f is continuous on $[a, b]$, $f(a)$ is positive and $f(b)$ is negative, f has more than one root in $[a, b]$.

2. Consider the function $f(x) = x^2 - 2.6x - 2.31$.

   (a) What *three* things tell us that the function has a root in the interval $[3, 4]$?

   (b) Find the derivative $f'(x)$, and use it to determine the values of x for which the function is increasing, and the values of x for which the function is decreasing. What is the function doing on the interval $[3, 4]$?

   (c) Let the interval $[a_1, b_1]$ be the interval $[3, 4]$. Give the next two intervals obtained using the bisection method, showing clearly all values computed and tests done to determine new endpoints.

   (d) Let $x_1 = 3$ and perform the next two iterations of Newton's method to obtain x_3, showing how you do it. **Round to five places past the decimal at the end of each computation.**

   (e) Use the quadratic formula to compute the root of f in the interval $[3, 4]$.

3. Do the following on the picture to the right, *using a straight-edge to draw all lines*, to illustrate/understand how Newton's method works. The curve is the graph of a function $f(x)$.

   (a) Draw a vertical line from x_1 to the curve, and label the point where it intersects the curve with its coordinates.

   (b) Draw a tangent line to the curve at the point you obtained in (a). Its x-intercept is x_2; label it.

   (c) Repeat two more times to show how x_3 and x_4 are found.