**Math 451** FINAL EXAM, "TAKE-HOME" PART **Due at 9 AM Tuesday, June 10th**

The purpose of this project is to learn how to do exponential regression, and compare results of quadratic and exponential regression for the automobile weight versus miles per gallon data. This will be broken into two parts: First you will explore how exponential regression works "by hand," then you will write some code to perform exponential regression, compute its error, and plot the exponential curve along with the data.

**You should work on this by yourself. You may use any other references you can find, and you may ask me questions (that I may or may not answer!). Otherwise you should be working on your own.**

### How Exponential Regression Works

**You will turn in the results of this part on paper!** Do your plotting and computations in the command window (or write a script file todo it) and transfer results to paper.

1. Plot the data points $(1, 0.7)$, $(2, 1.3)$, $(3, 3.2)$ on a grid that goes from $x = 0$ to $x = 4$. Sketch the graph neatly, accurately and reasonably sized (maybe $3\times3$ inches). There should be a scale and labels on each axis. *I ran into a problem when doing this, but a* `clear all` *fixed it...*

We wish to create an equation of the form $y = Ae^{kx}$, where $A$ and $k$ are constants, that reasonably approximates the data. The general idea is to plug the data points in for $x$ and $y$ to get a system of equations, but that system isn't linear so matrix methods can't be used to solve it. The trick is to take the natural log of both sides and apply the property of logarithms $\log_a uv = \log_a u + \log_a v$ and the fact that $\ln e^u = u$:

$$y = Ae^{kx} \quad \Longrightarrow \quad \ln y = \ln(Ae^{kx}) \quad \Longrightarrow \quad \ln y = \ln A + \ln e^{kx} \quad \Longrightarrow \quad \ln y = \ln A + kx \tag{1}$$

Comparing the last equation with $y = a_0 + a_1 x$ that we used for linear regression, you can see that we need to consider then values of $\ln y$ rather then the $y$ values from our data, and the values $\ln A$ and $k$ play the roles of $a_0$ and $a_1$. So you simply need to do linear regression with the $(x, \ln y)$ pairs from your data to find the values of $\ln A$ and $k$. You will then use the fact that $e^{\ln u} = u$ to determine $A$ and you'll have what you want! **NOTE:** In *MATLAB* (and most other computer applications), $e^u$ is obtained by `exp(u)` and the natural logarithm is `log`.

2. (a) Write the system of equations for the data using the final form of (1) above, but with the $\ln y$ values on the right side.

   (b) Give the matrix $A$ and vectors $\mathbf{x}$ and $\mathbf{b}$ for the matrix form $A\mathbf{x} = \mathbf{b}$ of the system. **Write each separately, labeled with what they are!**

   (c) Find the least-squares solution to the system, remembering that the *MATLAB* `A\b` will do that for you. **Write the solution down, with each component labeled as what it is from the final equation of (1).**

3. Determine $A$ and then plot the graph of $y = Ae^{kx}$ from $x = 0$ to $x = 4$ together with the data points. Sketch the curve onto your previously drawn graph.

4. The error in this case must be computed differently than using $A\mathbf{x} - \mathbf{b}$. For each $x$ data value you need to use your exponential equation to find the corresponding $y$ value. Make those new $y$ values components of a vector. The error is then the 2-norm of the difference between that vector and the vector of $y$ data values.

5. **Turn this written work in by noon Tuesday, June 10th at my office**.

### The Comparison Between Exponential and Quadratic Regression

1. For this you will need your file *yourfirstname_regr*. Rename it *yourfirstname_fa*. You may use my file *gregg_regr* instead, which you can find under 6/5 on the schedule or on the page of files. *If you use my file you will need to import the weight and miles per gallon vectors into the workspace.*

2. I would suggest that you first add some lines of code to determine the error of the linear regression by the method I described in Exercise 4 of the first part. *That error should be the same as the error that you computed from* $A\mathbf{x} - \mathbf{b}$. If not, fix it befroe going on.

3. Since your file already has linear and quadratic regression, you should be able to modify it so that the first regression is exponential and the second is still quadratic. Do that.

4. I would like two things for the output: **Side-by-side** graphs of the two regression curves with the data points, and the error (*just error, not the error vector*) for each method. Note the difference in finding error for the exponential regression that I described previously. To get a reminder/example of how to plot two graphs together, do a search for *matlab subplot*. Of course each graph should be labeled appropriately!

5. When the file is done, e-mail it to me **by 9 AM Tuesday, June 10th**. **NOTE:** Your file needs to run when I try it. I will have the *Excel* spreadsheet with the weight and mpg data in the folder that your file will be in when I try to run it, and I will also have the two vectors in the workspace, so your file needs to run under one of those two conditions.