

The following exercises are the sort of thing you could expect on the paper-and-pencil part of Exam 1. You will be able to use your calculator and one sheet of notes for the exam.

1. Consider a script file with the following lines in it:
- ```

for k=1:3
 a(k)=k^2;
end

```
- What should the output be if you ran it as is?
  - What should the output be if you removed the semicolon on the second line and ran it?
  - If you added a last line with just the letter `a` on it (with the semicolon on line two), what should the output be?
  - If instead you added a last line with just `a(k)` on it (again with the semicolon on line two), what should the output be?

Check your answers to this exercise by writing and running such scripts.

2. Suppose that you had been working for a while with various scripts that named a sequence `a`, then you ran the script given in Exercise 1 with a last line of `a` added and you got the following output:

```

a = 1 4 9 16 25

```

Why is this happening/how can it be fixed?

3. There is a function called the **floor function** that is quite useful for certain things. What the function basically does is round a number down (meaning left on the number line) to the nearest whole number, and the notation for the floor of a number  $x$  is  $\lfloor x \rfloor$ . Some examples are

$$\lfloor 4.7 \rfloor = 4, \quad \lfloor 3\frac{1}{2} \rfloor = 3, \quad \lfloor 18.1 \rfloor = 18, \quad \lfloor 5 \rfloor = 5, \quad \lfloor -3.4 \rfloor = -4$$

One use for this function is to test divisibility. For example, a number  $x$  is (“evenly”) divisible by three if, and only if,  $3\lfloor x/3 \rfloor = x$ . (You might wish to ponder this a little!) In *MATLAB* the floor function is simply `floor` - use this to determine the output of the script below. The double equal signs indicate that we are asking if the two things are equal, rather than assigning a value.

```

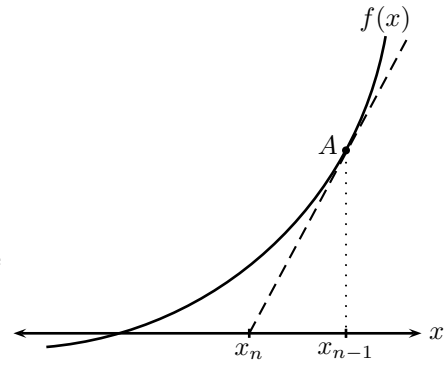
for k=1:5
 if 3*floor(k/3)==k
 a(k)=k+3;
 else
 a(k)=k;
 end
end
a

```

4. Suppose that you are looking for a root using the bisection method, and the actual value of the root to five places past the decimal is 1.23147. If  $[a_1, b_1]$  is the interval  $[0, 3]$ , give the intervals  $[a_2, b_2]$  through  $[a_5, b_5]$ .

5. Use the picture to the right to derive the formula for Newton's method. Use the following hints if necessary:

- Give the coordinates of point  $A$ .
- Give the slope of the dashed line in terms of the derivative of  $f$ .
- Give the slope of the dashed line in terms of  $f$  itself.
- Set your two expressions for the slope equal to each other and solve for  $x_n$ , showing clearly how you do it.



6. The code to the right is a portion from a script that performs the bisection method for finding a root of a function  $f$ .

- What is the stopping condition if we want to continue until the difference between  $a_k$  and  $b_k$  is less than 0.001?
- Fill in the blank spots with the lines needed there, based on the condition in the `if` statement.
- Rewrite what the code would look like if the `k=k+1` statement was the first line in the while loop, instead of being where it is.

```
while stopping condition
 m = (a(k)+b(k))/2;
 if f(m)*f(b(k))<0

 else

 end
 k=k+1
end
```

7. Now consider the script shown to the right.

- What will the output of the script be if we run it as is?
- What line would we add to fix the problem, and where would we put it?

```
clear all
a(1)=3;
k=1;
while a(k)<25
 a(k+1)=2*a(k);
end
a
```

8. Suppose that you are trying to solve the equation  $\sin x = e^{-x}$  in the interval  $[0, 2]$ . Show clearly how to find the next two intervals  $[a_2, b_2]$  and  $[a_3, b_3]$ , including all of the computations that would be needed for a machine to do this. Use some words like "First find ..., then compute... . If ... then ..., other wise ...". **Your calculator needs to be in radians!**

9. Recall that the formula for Newton's method is  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ .

Suppose that you are again trying to solve  $\sin x = e^{-x}$ . Use Newton's method with  $x_1 = 0$  to find  $x_2$  and  $x_3$ , showing clearly how you do it. Round your final answer and any intermediate computations to four places past the decimal, using all four places for any further computations. Check your answer to see if it comes close to solving the equation - it should.