

This is due at the start of class on Monday, March 6th.

1. The table to the right gives the values of a joint probability function  $f$  of two discrete random variables  $X$  and  $Y$ . Use it to answer each of the following. **Give your answers in exact form!**

		$x$		
		$f(x, y)$	0	1
$y$	0	$\frac{4}{16}$	$\frac{5}{16}$	$\frac{3}{16}$
	1	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

- (a) Give the distributions  $g(x)$  and  $w(y | 2)$ .

$x :$

$y :$

$g(x) :$

$w(y | 2) :$

- (b) Give  $P(X = 1 \text{ or } Y = 0)$  in terms of  $f$ ,  $g$  and  $h$ , then give its value. **Write your answer as a string, beginning with  $P(X = 1 \text{ or } Y = 0)$  and utilizing equal signs.**

- (c) Give  $P(X \leq Y)$  as two different summations, then give its value. **Again write your answer as a string. I'm going to quit saying this, but you need to do it every time.**

- (d) Give  $P(X + Y \geq 1)$  as two different summations, then write it in terms of  $f$  and using Theorem 1.3, then give its value.

- (e) Find the covariance  $\sigma_{XY}$ , showing clearly how you get it.

2. (a) Compute each of the following integrals, **getting your answers in exact form.**

$$\int_1^8 \int_0^2 x^2 y \, dy \, dx = \qquad \int_1^8 x^2 \, dx = \qquad \int_0^2 y \, dy =$$

- (b) Find the product of your answers to parts (b) and (c) of the previous exercise. What do you notice about the result? (**Write me a brief sentence, or write a concise mathematical statement containing integrals.**)

3. Consider the joint probability density function  $f$  for two continuous random variables  $X$  and  $Y$  given to the right. For each of the following, make a rough sketch of the region to the right in the space provided. Then give two iterated integrals whose values are the desired probability, then compute the probability. **As usual, connect everything in a string of equal expressions.**

$$f(x) = \begin{cases} 2e^{-2x-y} & \text{for } x \geq 0, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find  $P(X \geq 1, Y \leq 2)$ .

- (b) Find  $P(2X + 3Y \leq 6)$ .

- (c) Find  $P(Y \leq X - 1)$ . **Sketch the region carefully!**