Math 465 ASSIGNMENT 10, WINTER 2017 **Name:** ______________________

This is due at the start of class on Monday, March 6th.

1. The table to the right gives the values of a joint probability function f of two discrete random variables X and Y. Use it to answer each of the following. **Give your answers in exact form!**

	$f(x,y)$	x: 0	1	2	
y	0	$\frac{4}{16}$	$\frac{5}{16}$	$\frac{3}{16}$	$\frac{12}{16}$
	1	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{1}{16}$	$\frac{4}{16}$
		$\frac{5}{16}$	$\frac{7}{16}$	$\frac{4}{16}$	

(a) Give the distributions $g(x)$ and $w(y \mid 2)$.

x:	0	1	2
$g(x)$:	$\frac{5}{16}$	$\frac{7}{16}$	$\frac{4}{16}$

y:	0	1
$w(y \mid 2)$:	$\frac{3}{4}$	$\frac{1}{4}$

(b) Give $P(X=1 \text{ or } Y=0)$ in terms of f, g and h, then give its value. **Write your answer as a string, beginning with** $P(X=1 \text{ or } Y=0)$ **and utilizing equal signs.**

$$P(X=1 \text{ or } Y=0) = g(1)+h(0)-f(1,0) = \frac{7}{16}+\frac{12}{16}-\frac{5}{16} = \frac{14}{16} = \frac{7}{8}$$

(c) Give $P(X \le Y \cancel{\ge 0})$ as two different summations, then give its value. **Again write your answer as a string. I'm going to quit saying this, but you need to do it every time.**

$$P(X \le Y) = \sum_{x=0}^{1}\sum_{y=x}^{1} f(x,y) = \sum_{y=0}^{1}\sum_{x=0}^{y} f(x,y) = \frac{7}{16}$$

(d) Give $P(X+Y \ge 1)$ as two different summations, then write it in terms of f and using Theorem 1.3, then give its value.

As in class: $$P(X+Y \ge 1) = \sum_{y=0}^{1}\sum_{x=1-y}^{2} f(x,y) = 1-f(0,0)$$

(e) Find the covariance σ_{XY}, showing clearly how you get it.

$$\mu_X = 0\left(\tfrac{5}{16}\right)+1\left(\tfrac{7}{16}\right)+2\left(\tfrac{4}{16}\right) = \tfrac{15}{16} \qquad \mu_Y = 0\left(\tfrac{12}{16}\right)+1\left(\tfrac{4}{16}\right) = \tfrac{4}{16} = \tfrac{1}{4}$$

$$E(XY) = (1)(1)\left(\tfrac{2}{16}\right)+(1)(2)\left(\tfrac{1}{16}\right) = \tfrac{4}{16} = \tfrac{1}{4}$$

$$\sigma_{XY} = E(XY)-\mu_X\mu_Y = \tfrac{1}{4}-\left(\tfrac{15}{16}\right)\left(\tfrac{1}{4}\right) = \tfrac{16}{64}-\tfrac{15}{64} = \boxed{\tfrac{1}{64}}$$

2. (a) Compute each of the following integrals, **getting your answers in exact form.**

$$\int_1^8 \int_0^2 x^2 y \, dy \, dx = \frac{1022}{3} \qquad \int_1^8 x^2 \, dx = \frac{511}{3} \qquad \int_0^2 y \, dy = 2$$

(b) Find the product of your answers to parts (b) and (c) of the previous exercise. What do you notice about the result? (**Write me a brief sentence, or write a concise mathematical statement containing integrals.**)

$$\int_1^8 \int_0^2 x^2 y \, dy \, dx = \left(\int_1^8 x^2 \, dx\right)\left(\int_0^2 y \, dy\right)$$

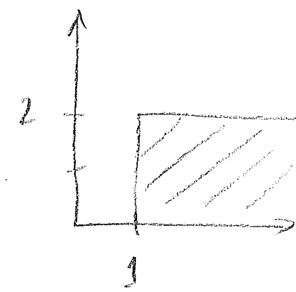
3. Consider the joint probability density function f for two continuous random variables X and Y given to the right. For each of the following, make a rough sketch of the region to the right in the space provided. Then give two iterated integrals whose values are the desired probability, then compute the probability. **As usual, connect everything in a string of equal expressions.**

$$f(x) = \begin{cases} 2\,e^{-2x-y} & \text{for } x \geq 0, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find $P(X \geq 1, Y \leq 2)$.

$$P(X \geq 1, Y \leq 2) = \int_1^\infty \int_0^2 2e^{-2x-y} \, dy \, dx$$

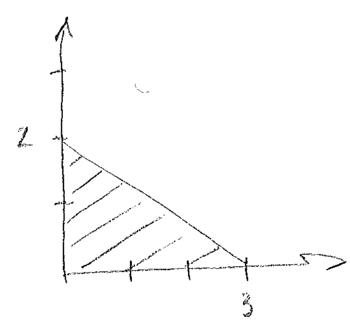
$$= \int_0^2 \int_1^\infty 2e^{-2x-y} \, dx \, dy = \frac{e^2-1}{e^4} \approx 0.117$$



(b) Find $P(2X + 3Y \leq 6)$.

$$P(2X + 3Y \leq 6) = \int_0^3 \int_0^{y=-\frac{2}{3}x+2} 2e^{-2x-y} \, dy \, dx$$

$$= \int_0^2 \int_0^{x=-\frac{3}{2}y+3} 2e^{-2x-y} \, dx \, dy = 0.798$$



(c) Find $P(Y \leq X - 1)$. **Sketch the region carefully!**

$y = x-1$

$$P(Y \leq X - 1) = \int_1^\infty \int_0^{x-1} 2e^{-2x-y} \, dy \, dx$$

$$= \int_0^\infty \int_{y+1}^\infty 2e^{-2x-y} \, dx \, dy = 0.045$$