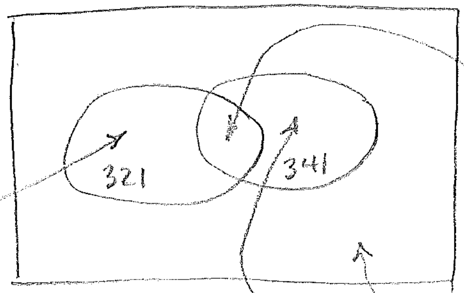
Math 465, Assignment 1 W17

(1) Start with this:

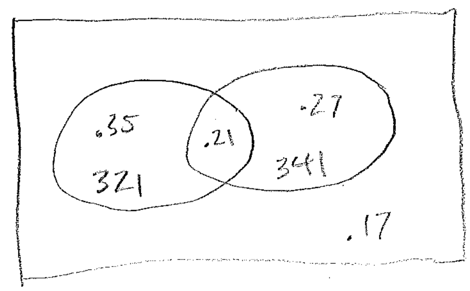


(1) Because the prob of 321 but not 341 is 0.35, that goes here

(2) Because the total prob of 321 is 0.56, $0.56-0.35=0.21$ goes here.

(3) The prob of having taken neither 321 nor 341 is 0.17, which goes here.

(4) The only unknown probability is here, and it must be one minus the sum of all the other probabilities, or
$1-(0.35+0.21+0.17) = 1-0.73=0.27$.
We now have this

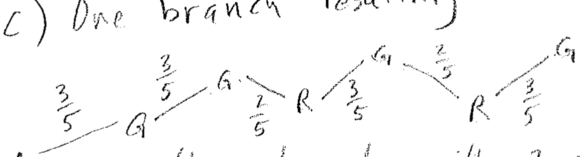


Now we see that (a) is 0.21 and (b) is $0.35+0.17=0.52$. For (c) we compute $\frac{0.21}{0.48}$, the proportion of the 341 students that have also taken 321.
For (d) we need the proportion of students who have not taken 321 that have taken 341, which is $\frac{0.27}{0.27+0.17} = \frac{0.27}{0.44}$

(5) a) $2\cdot2\cdot2\cdot2\cdot2\cdot2 = 2^6$ because there are two color choices for each of the six marbles.

b) By Theorem 1.9, $\frac{6!}{2!\,4!} = \frac{6\cdot5}{2} = 15$

c) One branch resulting in two reds might be



so $P(GGRGRG) = \frac{3\cdot3\cdot2\cdot3\cdot2\cdot3}{5\cdot5\cdot5\cdot5\cdot5\cdot5} = \frac{324}{15625}$

Any other branch with 2 red and 4 green will be the same value, with the 3's and 2's rearranged. Since there are 15 such branches,

d) The probability of exactly two reds is $15\left(\frac{2\cdot2\cdot3\cdot3\cdot3\cdot3}{5\cdot5\cdot5\cdot5\cdot5\cdot5}\right) = \frac{972}{3125}$

e) $P(\text{at least 2 red}) = 1 - P(\text{zero or one red})$

$= 1 - [P(\text{zero red}) + P(\text{1 red})]$

continued on next page

Math 465, Assignment 1, continued

W17

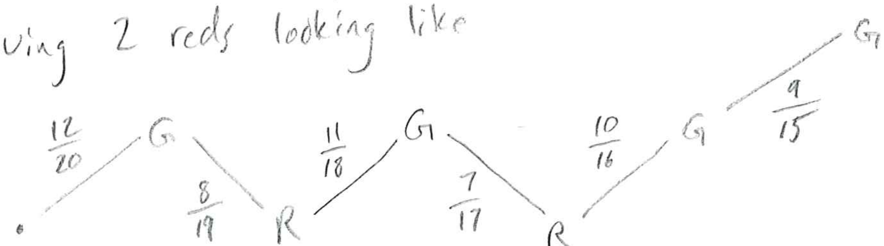
(5) e) continued

There is only one way to get no red, with probability $\frac{3^6}{5^6} = \left(\frac{3}{5}\right)^6$

When getting exactly one red, there are 6 places the red could be, so there are 6 ways for that to happen. The probability of one red and 5 green in any one of the six ways is $\frac{2}{5}\cdot\left(\frac{3}{5}\right)^5$, so the total probability of exactly one red is $6\left[\frac{2}{5}\left(\frac{3}{5}\right)^5\right]$. Then P(at least 2 red) is

$$1 - \left[\left(\frac{3}{5}\right)^6 + 6\cdot\frac{2}{5}\left(\frac{3}{5}\right)^5\right] = 1 - \left(\frac{3}{5}\right)^5\left[\frac{3}{5} + 6\cdot\frac{2}{5}\right]$$
$$= 1 - \left(\frac{3}{5}\right)^5\left(\frac{15}{5}\right)$$
$$= 1 - \frac{3^6}{5^5} = \frac{5^5 - 3^6}{5^5} = \frac{2396}{3125}$$

f) This is the same question as (d), but with a branch having 2 reds looking like



The probability for any branch with two red (and, consequently) four green is $\frac{8\cdot 7\cdot 12\cdot 11\cdot 10\cdot 9}{20\cdot 19\cdot 18\cdot 17\cdot 16\cdot 15}$. There are again 15 branches with two red and four green, so the probability of exactly two red is

$$15\left(\frac{8\cdot 7\cdot 12\cdot 11\cdot 10\cdot 9}{20\cdot 19\cdot 18\cdot 17\cdot 16\cdot 15}\right) = \frac{7\cdot 11\cdot 3}{2\cdot 19\cdot 17} = \frac{231}{646}$$

(1)(c) $P(321) = 0.56$, $P(341) = 0.48$

$P(321 \cap 341) = 0.21 \neq P(321)P(341) \approx 0.27$