

This assignment is due at the start of class on Monday, January 30th

1. A *discrete* random variable  $X$  has the cumulative distribution function  $F(x)$  shown below and to the right.

$$\begin{array}{l}
 x : \\
 f(x) :
 \end{array}
 \qquad
 F(x) = \begin{cases}
 0 & \text{for } x < 0 \\
 \frac{1}{12} & \text{for } 0 \leq x < 1 \\
 \frac{6}{12} & \text{for } 1 \leq x < 2 \\
 \frac{10}{12} & \text{for } 2 \leq x < 3 \\
 1 & \text{for } x \geq 3
 \end{cases}$$

- (a) Give each of the following probabilities based on  $F$ .

$$P(X \leq 2) = \underline{\hspace{2cm}} \qquad P(X \geq 2) = \underline{\hspace{2cm}} \qquad P(X = 1) = \underline{\hspace{2cm}}$$

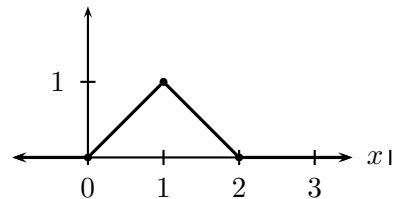
$$P(X = 1\frac{1}{2}) = \underline{\hspace{2cm}} \qquad P(X \leq 1\frac{1}{2}) = \underline{\hspace{2cm}} \qquad P(1 < X \leq 3) = \underline{\hspace{2cm}}$$

- (b) Give the probability distribution function  $f$  in the space above and to the left.

2. Give the value of  $c$  for which  $f(x) = \frac{c}{x^2}$ ,  $x = 1, 3, 5$  is a discrete probability distribution. Show how you obtain it.

3. The probability density function, and its graph, for a **continuous** random variable  $X$  is shown below. Use it to find the things that follow. You can integrate, or simply use some geometry.

$$f(x) = \begin{cases}
 x & \text{for } 0 \leq x < 1 \\
 2 - x & \text{for } 1 \leq x \leq 2 \\
 0 & \text{elsewhere}
 \end{cases}$$



(a)  $P(X > \frac{1}{2}) = \underline{\hspace{2cm}}$

(b)  $P(X = \frac{1}{2}) = \underline{\hspace{2cm}}$

(c)  $P(\frac{1}{4} < X < \frac{3}{4}) = \underline{\hspace{2cm}}$

(d)  $F(\frac{5}{2}) = \underline{\hspace{2cm}}$

4. An experiment consists of rolling a single die, so  $S = \{1, 2, 3, 4, 5, 6\}$ . The random variable  $X$  assigns to each outcome the number of letters it has when spelled as a word. For example,  $X(5) = 4$  since the word *five* has four letters.

(a) Fill in the blanks:  $X(1) = \underline{\hspace{1cm}}$ ,  $X(2) = \underline{\hspace{1cm}}$ ,  $X(3) = \underline{\hspace{1cm}}$

(b) Give the range of  $X$ , using appropriate notation:

(c) Give the event  $A$  (as a subset of the sample space) corresponding to  $X = 4$ :

(d) Give the event  $B$  (as a subset of the sample space) corresponding to  $X \geq 4$ :

(e) Fill in the blanks:  $f(5) = P(X = 5) = \underline{\hspace{1cm}}$ ,  $f(3) = P(\underline{\hspace{1cm}}) = \underline{\hspace{1cm}}$ ,

$f(\underline{\hspace{1cm}}) = P(X = 4) = \underline{\hspace{1cm}}$ ,  $f(2) = P(\underline{\hspace{1cm}}) = \underline{\hspace{1cm}}$

(f) Fill in the blanks:  $F(4) = P(X \leq 4) = \underline{\hspace{1cm}}$ ,  $F(7) = P(\underline{\hspace{1cm}}) = \underline{\hspace{1cm}}$ ,

$F(\underline{\hspace{1cm}}) = P(X \leq 3.5) = \underline{\hspace{1cm}}$ ,  $F(2) = P(\underline{\hspace{1cm}}) = \underline{\hspace{1cm}}$

(g) Give the probability distribution function  $f$  for this random variable **on the axes below and to the left**. Be sure to label the horizontal axis with all integers from the smallest value in  $\text{Ran}(X)$  to the largest value in  $\text{Ran}(X)$ .



(h) Give the cumulative probability distribution function  $F$  **on the axes above and to the right**.