Math 465 ASSIGNMENT 3 **Name:** ______________________________

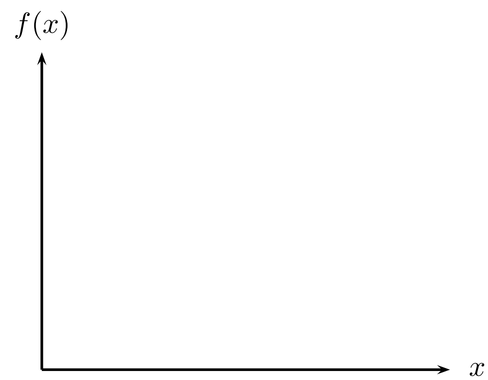
This assignment is due at the start of class on Friday, February 3rd

1. A *continuous* random variable X has the probability density function $f(x)$ shown below and to the right.

$$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ 2c & \text{for } 0 \le x < 2 \\ 3c & \text{for } 2 \le x < 4 \\ c & \text{for } 4 \le x < 8 \\ 0 & \text{for } x \ge 8 \end{cases}$$

(a) Determine the value of c for this to be a probability density function, showing how you do it.

$c =$ __________



(b) Sketch a graph of f with reasonably accurate scales on each axis on the axes to the right.

(c) Give each of the following probabilities based on f.

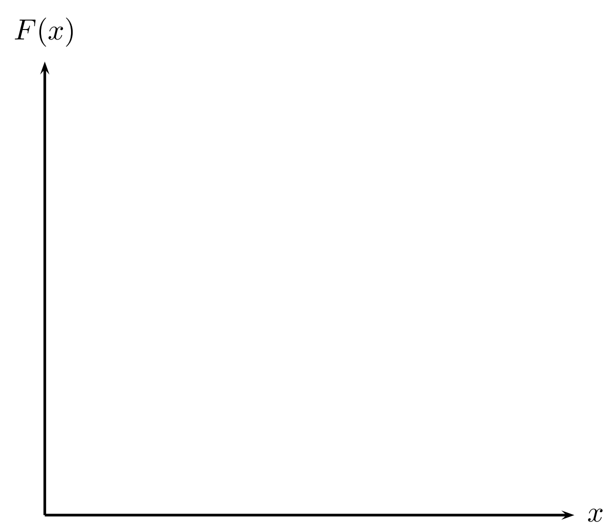
$P(X \le 3) =$ _______ $P(X \ge 5) =$ _______

$P(1 \le X \le 7) =$ _______ $P(X = 4) =$ _______

(d) In the space to the right, sketch a graph of **approximately** what you expect the graph of the corresponding cumulative probability function F to look like.

(e) Give the cumulative probability function F below and to the left, and give its graph below and to the right. (Again, make your graph reasonably accurate.) **On extra paper, show work for how you are obtaining F.**

$$F(x) = \begin{cases} & \end{cases}$$



There is more on the back!

2. Write in the probability density function f from Exercise 1 (with numbers, rather than multiples of c) in the space below and to the right. Then find the expected value $\mu = E(X)$ and variance σ^2 for it, **using Theorem 2.9 to find the variance. Show all integrals used and their values on your extra paper or in the space below. You may use your calculator or an online tool to calculate the integrals.**

   (a) $E(X) =$ __________

   (b) $E(X^2) =$ __________ (See Theorem 2.9 for what this is and why you need it.)

   (c) $\sigma^2 =$ __________

3. Note that the probability density function from exercise is nonzero on three intervals. We can create a **discrete** probability function by "putting" the probability of each individual interval at the mean (center) of that interval. We'll call this new function g - give it to the right.

   x :

   $g(x)$:

   (a) Find the expected value for this distribution - you should notice something special about it! Show work on your extra paper or in the space below, indicating how you obtain your answer.

   (b) Find the variance for this distribution. I'm not sure whether you will see the same thing as you saw for the expected value - I'll find out when I do it!