Math 465, Assignment 6 8 points +2 W17

(1) a) $\int x^5 e^x dx = x^5 e^x - 5\int x^4 e^x dx$ $\quad u = x^5 \quad dv = e^x dx$, $\quad du = 5x^4 dx \quad v = e^x$

$+\frac{1}{2}$

b) $\int_0^{\frac{\pi}{2}} x\cos x dx = x\sin x\Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x dx$ $\quad u = x \quad dv = \cos x dx$, $\quad du = dx \quad v = \sin x$

$$= \left(\frac{\pi}{2}\sin\frac{\pi}{2} - 0\right) + \cos x\Big|_0^{\frac{\pi}{2}}$$

+1

$$= \frac{\pi}{2} + \cos\frac{\pi}{2} - \cos 0$$

$$= \frac{\pi}{2} - 1$$

c) $\int_0^b x^n e^{-x} dx = -x^n e^{-x}\Big|_0^b + n\int_0^b x^{n-1} e^{-x} dx$ $\quad u = x^n \quad dv = e^{-x} dx$, $\quad du = nx^{n-1} \quad v = -e^{-x}$

$$= -b^n e^{-b} + 0 + n\int_0^b x^{n-1} e^{-x} dx$$

$$= -b^n e^{-b} + n\int_0^b x^{n-1} e^{-x} dx$$

+1½

+2

$$\int_0^\infty x^n e^{-x} dx = \lim_{b\to\infty}\left[-b^n e^{-b} + n\int_0^b x^{n-1} e^{-x} dx\right]$$

$$= n\int_0^\infty x^{n-1} e^{-x} dx$$

(2) a) $\int_{-1}^{3} (4x+12)\, dx = \int_{-2}^{6} [4(u-3)+12]\, du$ $\quad u = x+3$, $x = u-3$, $du = dx$, $x=-1 \Rightarrow u=-4$, $x=3 \Rightarrow u=0$

+1

$$= \int_2^6 4u\, du$$

$$= 4\int_2^6 u\, du$$

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(2) b) $$\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{1}{x^2}\,dx = -\int_4^2 du = \int_2^4 du$$

$u = \frac{1}{x} = x^{-1}$

$du = -x^{-2}\,dx$

$-du = \frac{1}{x^2}\,dx$

$x = \frac{1}{4} \to u = 4$

$x = \frac{1}{2} \to u = 2$

+1

c) $$\int_0^1 e^{-4x^2}\,dx = \frac{1}{2}\int_0^2 e^{-u^2}\,du$$

$u = 2x$ $\quad x = 0 \Rightarrow u = 0$

$du = 2dx$ $\quad x = 1 \Rightarrow u = 2$

$dx = \frac{1}{2}du$

+1

(3)

$$f(0) = \int_0^2 x^0\,dx = \int_0^2 dx = x\Big|_0^2 = 2$$

$$f(1) = \int_0^2 x\,dx = \frac{1}{2}x^2\Big|_0^2 = 2$$

$$f(2) = \int_0^2 x^2\,dx = \frac{1}{3}x^3\Big|_0^2 = \frac{8}{3}$$

$+1\frac{1}{2}$