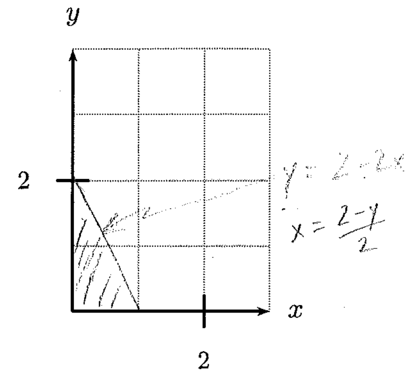
Math 465 ASSIGNMENT 9, WINTER 2017 **Name:** ________________________

Give all answers NEATLY on this paper. If you know you have trouble with neatness, you may wish to (that is, I want you to!) work things out on other paper and put your final answers on here.

1. You wish to integrate the function $f(x,y) = x + 2y$ over the region in the first quadrant bounded by $y = 2 - 2x$, $x = 0$ and $y = 0$.

   (a) Sketch the region on the grid to the right and shade it.

   (b) Give two iterated integrals that could be used to determine the value of the desired integral. **Since they should be equal, connect them with equal signs.**



$$\int_0^1\int_0^{2-2x}(x+2y)\,dy\,dx = \int_0^2\int_0^{\frac{2-y}{2}}(x+2y)\,dx\,dy$$

   (c) Compute the value of **one** of your iterated integrals from part (b) **by hand**, showing your work clearly in a horizontal string connected with equal signs.

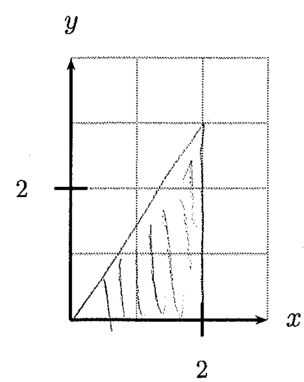
$$\int_0^1\int_0^{2-2x}(x+2y)\,dy\,dx = \int_0^1\left[xy+y^2\right]_0^{2-2x}dx = \int_0^1\left[2x-2x^2+(2-2x)^2\right]dx = \int_0^1(2x^2-6x+4)\,dx$$
$$= \frac{2}{3}x^3-3x^2+4x\Big|_0^1 = 1\frac{2}{3}$$

   (d) Use the *Wolfram Alpha Double Integral Calculator* or your calculator to evaluate both integrals from (b). They should be equal to each other and to your answer to (c). If not, find and correct any errors.

2. You wish to integrate the function $f(x,y) = xy$ over the region in the first quadrant bounded by $y = \frac{3}{2}x$ and $x = 2$.

   (a) Sketch the region on the grid to the right and shade it.

   (b) Give two iterated integrals that could be used to determine the value of the desired integral. **Since they should be equal, connect them with equal signs.**



$$\int_0^3\int_{\frac{2}{3}y}^{2} xy\,dx\,dy = \int_0^2\int_0^{\frac{3}{2}x} xy\,dy\,dx = \frac{9}{2} = 4.5$$

   (c) Use the *Wolfram Alpha Double Integral Calculator* or your calculator to evaluate both integrals from (b). They should be equal to each other - if not, find and correct any errors.

3. The joint probability density function shown to the right is for two continuous random variables X and Y. Use it for each of the following.

$$f(x) = \begin{cases} \frac{1}{26}(x^2 + 2y) & \text{for } (x, y) \in [0, 2] \times [0, 3] \\ 0 & \text{otherwise} \end{cases}$$

(a) Find $P(\frac{1}{2} \le X \le 1, 1 \le Y \le 2)$. Show the integral used and use technology to compute it.

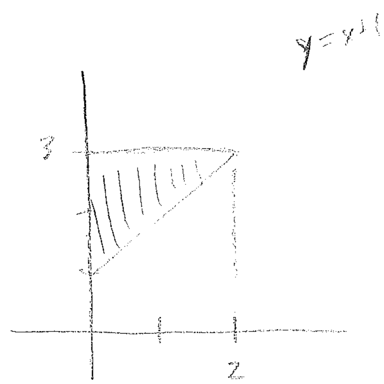
$$P(\tfrac{1}{2} \le X \le 1, 1 \le Y \le 2) = \int_{\frac{1}{2}}^{1} \int_{1}^{2} \frac{1}{26}(x^2+2y)\,dy\,dx = \int_{1}^{2} \int_{\frac{1}{2}}^{1} \frac{1}{26}(x^2+2y)\,dx\,dy = \frac{43}{624}$$

$$= 0.0689$$

(b) Sketch the region where $Y \ge X + 1$ and then find $P(Y \ge X + 1)$.

$$P(Y \ge X+1) = \int_{x=0}^{2} \int_{y=x+1}^{3} \frac{1}{26}(x^2+2y)\,dy\,dx = \frac{16}{39} = 0.4103$$

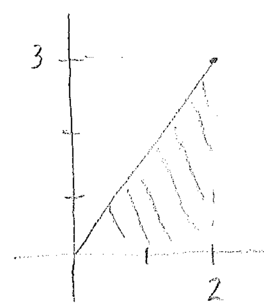
$$= \int_{1}^{3} \int_{0}^{y-1} \frac{1}{26}(x^2+2y)\,dx\,dy = \frac{16}{39} = 0.4103$$



(c) Sketch the region where $Y \le \frac{3}{2}X$ and then find $P(Y \le \frac{3}{2}X)$.

$$P(Y \le \tfrac{3}{2}X) = \int_{x=0}^{2} \int_{y=0}^{\frac{3}{2}x} \frac{1}{26}(x^2+2y)\,dy\,dx = \frac{6}{13} = 0.4615$$

$$= \int_{y=0}^{3} \int_{x=\frac{2}{3}y}^{2} \frac{1}{26}(x^2+2y)\,dx\,dy = \frac{6}{13} = 0.4615$$



(d) Give the marginal distributions $g(x)$ and $h(y)$, showing clearly how you obtain them.

$$g(x) = \int_0^3 \frac{1}{26}(x^2+2y)\,dy = 3x^2+9 \qquad h(y) = 4y + \frac{8}{3}$$

(e) Give the conditional distribution $v(x \mid y) = \dfrac{\frac{1}{26}(x^2+2y)}{4y+\frac{8}{3}} = \dfrac{3(x^2+2y)}{312y+208}$

(f) Give the conditional distribution $w(y \mid 1) = \dfrac{\frac{1}{26}(1+2y)}{12} = \dfrac{1}{312}(1+2y)$