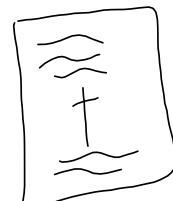


$$f(x) = x^2$$

$$f: \mathbb{R} \rightarrow \mathbb{R}$$



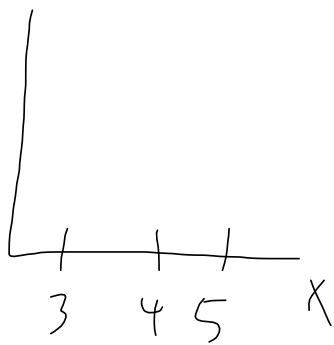
$$5 \rightarrow 25$$

x	$g(x)$
1	π
2	e
3	17
4	-5

$$g: \{1, 2, 3, 4\} \rightarrow \mathbb{R}$$

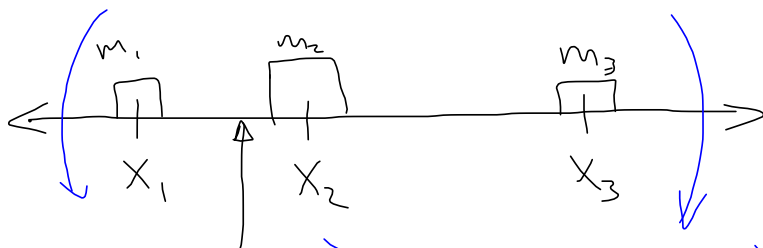
$$x: \{1, 2, 3, 4\}$$

$$g(x): \{\pi, e, 17, -5\}$$



$$\text{Ran}(\bar{X}) = \{3, 4, 5\}$$

$$.10(79) + .30(91) + .60(82)$$



$$m_1(\bar{x} - x_1) = m_2(x_2 - \bar{x}) + m_3(x_3 - \bar{x})$$

$$0 = m_2(x_2 - \bar{x}) + m_3(x_3 - \bar{x}) - m_1(\bar{x} - x_1)$$

$$0 = m_2x_2 - m_2\bar{x} + m_3x_3 - m_3\bar{x} + m_1x_1 - m_1\bar{x}$$

$$(m_1 + m_2 + m_3)\bar{x} = m_1x_1 + m_2x_2 + m_3x_3$$

$$\bar{x} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$\underbrace{m_1 + m_2 + m_3}_M$$

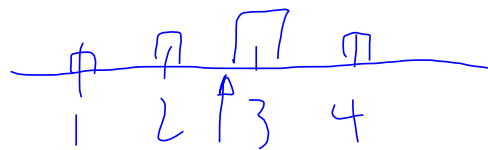
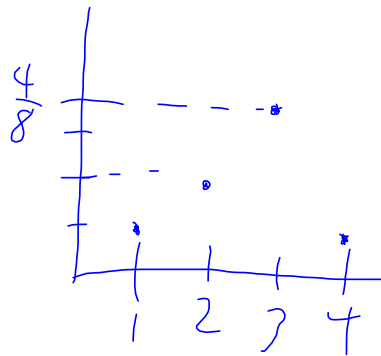
$$\bar{x} = \frac{m_1}{M}x_1 + \frac{m_2}{M}x_2 + \frac{m_3}{M}x_3$$

x	$f(x)$
1	$\frac{1}{8}$
2	$\frac{2}{8}$
3	$\frac{4}{8}$
4	$\frac{1}{8}$

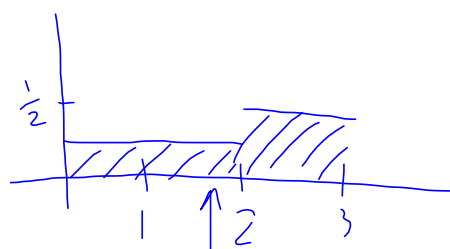
Expected value of X

is
$$\sum x f(x) = (1)\left(\frac{1}{8}\right) + (2)\left(\frac{2}{8}\right) + (3)\left(\frac{4}{8}\right) + 4\left(\frac{1}{8}\right)$$

$$= \frac{21}{8} = 2\frac{5}{8}$$



$$f(x) = \begin{cases} \frac{1}{4} & \text{for } 0 \leq x < 2 \\ \frac{1}{2} & \text{for } 2 \leq x < 3 \\ 0 & \text{otherwise} \end{cases}$$



$$\begin{aligned} \mu = E(X) &= \int_0^3 x f(x) dx = \int_0^2 \frac{1}{4} x dx + \int_2^3 \frac{1}{2} x dx \\ &= \frac{1}{2} + \frac{5}{4} = \frac{3}{4} \end{aligned}$$



Variance

$$\sigma^2 = E(X - \mu) = \sum (x - \mu) f(x)$$
$$\sigma^2 = E(X^2) - \underbrace{[E(X)]^2}_{\mu^2} \quad E(X^2) = \sum x^2 f(x)$$
$$E(X^2) = \int x^2 f(x) dx$$

x	f(x)
1	$\frac{1}{8}$
2	$\frac{2}{8}$
3	$\frac{4}{8}$
4	$\frac{1}{8}$

$$E(X) = 2\frac{5}{8} = \frac{21}{8}$$

$$E(X^2) = \sum x^2 f(x)$$

$$= (1)^2\left(\frac{1}{8}\right) + (2)^2\left(\frac{2}{8}\right) + (3)^2\left(\frac{4}{8}\right) + (4)^2\left(\frac{1}{8}\right)$$
$$= \frac{61}{8}$$

$$\sigma^2 = \frac{61}{8} - \left(\frac{21}{8}\right)^2$$
$$= \frac{488}{64} - \frac{441}{64} = \frac{47}{64}$$

