Let $a>0$ be a constant. Find

$$\int_0^\infty xe^{-ax}\,dx.$$

$$\boxed{\int_0^\infty xe^{-ax}\,dx = \frac{1}{a^2}}$$

if $a>0$

if $a>0$

$$\boxed{f(x) = \frac{1}{\beta} e^{-\frac{x}{\beta}} \quad x > 0 \quad (f(x) = 0 \text{ if } x \leq 0)}$$

$$E(X) = \int_{-\infty}^{\infty} x f(x)$$

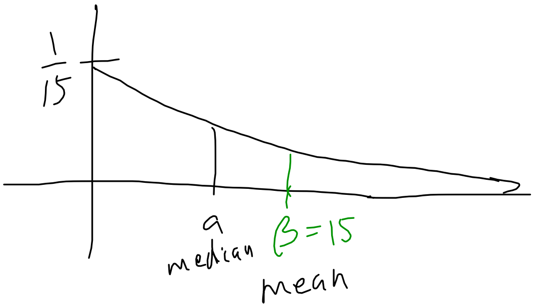
$$E(X) = \int_0^{\infty} x \frac{1}{\beta} e^{-\frac{1}{\beta}x} dx = \frac{1}{\beta} \int_0^{\infty} x e^{-\frac{1}{\beta}x} dx = \frac{1}{\beta} \cdot \frac{1}{\left(\frac{1}{\beta}\right)^2} = \beta$$

Exp dist, $\beta = 15$. $P(X \le 15) \stackrel{?}{=} \frac{1}{2}$

$$P(X \le 15) = F(15) = 1 - e^{-\frac{15}{15}} = 1 - e^{-1}$$

$$= 0.6321$$

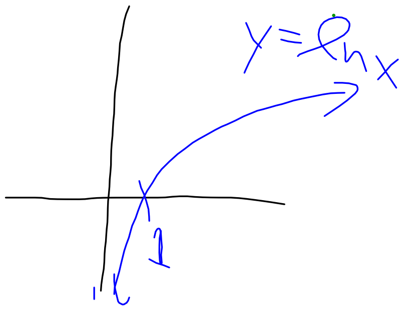
$f(x) = \frac{1}{15} e^{-\frac{x}{15}}$



$P(X < a) = \frac{1}{2}$

$P(X<a) = F(a) = 1 - e^{-\frac{a}{15}} = \frac{1}{2}$

$\frac{1}{2} = e^{-\frac{a}{15}}$



$\ln \frac{1}{2} = \ln\left(e^{-\frac{a}{15}}\right)$

$\ln \frac{1}{2} = -\frac{a}{15}$

$-15 \ln \frac{1}{2} = a$

$f(x) = x^2 - 5x$

$f(x) = \sin 3x$

$f(x) = \int_1^x e^{-2t}\,dt$

$f(x) = \int_0^5 xt\,dt$

$$F(s) = \int_0^\infty t^2 e^{-st}\,dt$$

For Friday, Do 4.7: 1-8
due

Skip 1b, for # four
They change 13 sets
of tires per hour

$$\underbrace{\int_0^b x e^{-ax}\,dx} = -\frac{1}{a} x e^{-ax}\Big|_0^b + \frac{1}{a}\int_0^b e^{-ax}\,dx \qquad u = x \qquad dv = e^{-ax}dx$$

$$= -\frac{b}{a}e^{-ab} - \frac{1}{a^2}e^{-ax}\Big|_0^b \qquad du = dx \qquad v = -\frac{1}{a}e^{-ax}$$

$$= -\frac{b}{a}e^{-ab} - \frac{1}{a^2}e^{-ab} + \frac{1}{a^2}$$

$$\int_0^\infty x e^{-ax}\,dx = \lim_{b\to\infty}\int_0^b x e^{-ax}\,dx = \lim_{b\to\infty}\left[-\frac{b}{a}e^{-ab} - \frac{1}{a^2}e^{-ab} + \frac{1}{a^2}\right] = \frac{1}{a^2}$$