

3 marbles are drawn, without replacement, from an urn containing 3 yellow marbles and two blue. Let X be the number of yellow marbles drawn, and Y the number of blue drawn. Make a joint prob dist table.

		X		
f(x,y)		1	2	3
Y	0	0	0	$\frac{3}{5} \cdot \frac{2}{4} \cdot \frac{1}{3} (1)$
	1	0	$\frac{36}{60}$	0
	2	$\frac{18}{60}$	0	0
		$\frac{18}{60}$	$\frac{36}{60}$	$\frac{6}{60}$

X and Y
are not
indep because

$$g(1)h(0) = \frac{6}{60} \cdot \frac{18}{60} = \frac{108}{3600} \neq 0 = f(1,0)$$

$P(A \cap B) = P(A)P(B) \implies A \text{ and } B \text{ are independent}$

X and Y are indep if $f(x,y) = g(x)h(y)$
for all $(x,y) \in \text{Ran}(X) \times \text{Ran}(Y)$

$$x: \quad 1 \quad 2 \quad 3$$
$$g(x): \quad \frac{18}{60} \quad \frac{36}{60} \quad \frac{6}{60}$$

$$v(x|y) = \frac{f(x,y)}{h(y)}$$

fixed

Give $v(x|2)$

$$x: \quad 1 \quad 2 \quad 3$$

$$v(x|2): \quad 1 \quad 0 \quad 0$$

Conditional distribution

$$w(y/x) = \frac{f(x,y)}{g(x)}$$

fixed

$$\int_0^{\infty} x^n e^{-x} dx \quad u = x^n \quad dv = e^{-x} dx$$

$$du = nx^{n-1} dx \quad v = -e^{-x}$$

$$\int_0^b x^n e^{-x} dx = -x^n e^{-x} \Big|_0^b + n \int_0^b x^{n-1} e^{-x} dx$$

$$= -b^n e^{-b} + n \int_0^b x^{n-1} e^{-x} dx$$

$$\int_0^{\infty} x^n e^{-x} dx = \lim_{b \rightarrow \infty} \left[\cancel{-b^n e^{-b}} + n \int_0^b x^{n-1} e^{-x} dx \right]$$

$$\int_0^{\infty} x^n e^{-x} dx = n \int_0^{\infty} x^{n-1} e^{-x} dx$$

$$\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx$$

$$\Gamma(1) = \int_0^{\infty} e^{-x} dx = 1$$

$$\Gamma(2) = \int_0^{\infty} x^{2-1} e^{-x} dx = \int_0^{\infty} x^1 e^{-x} dx = 1 \int_0^{\infty} x^0 e^{-x} dx = 1$$

$$\begin{aligned} \Gamma(\alpha) &= \int_0^{\infty} x^{\alpha-1} e^{-x} dx = (\alpha-1) \int_0^{\infty} x^{(\alpha-1)-1} e^{-x} dx \\ &= (\alpha-1) \Gamma(\alpha-1) \end{aligned}$$

$$\Gamma(1) = 1$$

$$\Gamma(x) = (x-1)\Gamma(x-1)$$

$$\Gamma(2) = 1$$

$$\Gamma(n) = (n-1)! \quad n=1, 2, 3, \dots$$

$$\Gamma(3) = (3-1)\Gamma(3-1) = 2\Gamma(2) = 2 \cdot 1$$

$$\Gamma(4) = (4-1)\Gamma(4-1) = 3\Gamma(3) = 3 \cdot 2 \cdot 1$$

$$\Gamma(5) = 4\Gamma(4) = 4 \cdot 3 \cdot 2 \cdot 1$$

$$P(Y=1) = \sum = \emptyset$$

$$P(\bar{X} \leq \bar{Y} - 1) = \sum_{X=} \sum_{Y=} = \sum_{Y=} \sum_{X=} =$$

