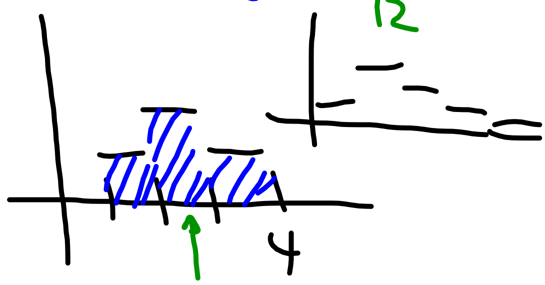
For a continuous random variable X:

$$f(x) = \begin{cases} \frac{1}{4} & \text{if } 1 \le x < 2 \\ \frac{1}{2} & \text{if } 2 \le x < 3 \\ \frac{1}{4} & \text{if } 3 \le x < 4 \\ 0 & \text{otherwise} \end{cases}$$

Find $E(X)$

and $\sigma^2 = \frac{7}{12}$

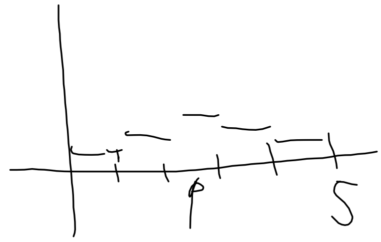


For another continuous random variable Y,

$$g(y) = \begin{cases} \frac{1}{9} \text{ if } 0 \le x < 1 \\ \frac{2}{9} \text{ if } 1 \le x < 2 \\ \frac{3}{9} \text{ if } 2 \le x < 3 \\ \frac{2}{9} \text{ if } 3 \le x < 4 \\ \frac{1}{9} \text{ if } 4 \le x < 5 \\ 0 \text{ otherwise} \end{cases}$$

Find $E(Y)$

and $\sigma^2 = \frac{17}{12}$



$$f(x)=\begin{cases} 0 & \text{for } x<0 \\ \frac{1}{2} & \text{for } 0\le x<1 \\ -\frac{1}{4}x+\frac{3}{4} & \text{for } 1\le x<3 \\ 0 & \text{for } x\ge 3 \end{cases}$$

$$\frac{9}{8} \qquad \frac{18}{8}-\frac{9}{8} \qquad \frac{9}{4}-\frac{9}{8}$$

$$F(x)=\begin{cases} 0 & x<0 \\ \frac{1}{2}x & 0\le x<1 \\ -\frac{1}{8}x^2+\frac{3}{4}x-\frac{1}{8} & 1\le x<3 \\ 1 & x>3 \end{cases}$$

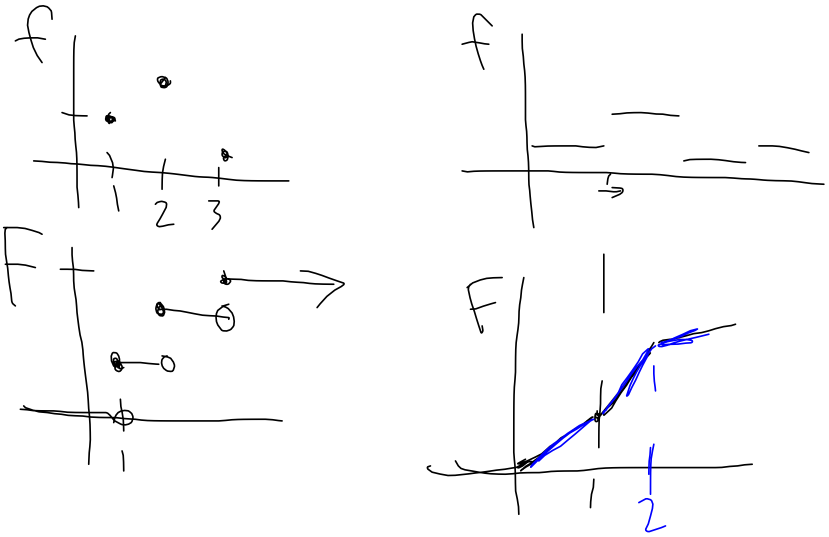
$$F(x) = \int_{-\infty}^{x} f(t)\,dt = \int_{0}^{1} \frac{1}{2}\,dt + \int_{1}^{x} \left(-\frac{1}{4}t + \frac{3}{4}\right) dt$$

$1 \leq x < 3$

$$= \frac{1}{2} + \left[-\frac{1}{8}t^2 + \frac{3}{4}t\right]_1^x$$

$$= -\frac{1}{8}x^2 + \frac{3}{4}x - \left(-\frac{1}{8} + \frac{3}{4}\right) + \frac{1}{2}$$

Discrete Continuous



$$F(x) = \begin{cases} & 1 \le x < 2 \\ & 2 \le x < 3 \\ & \end{cases}$$

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx = 2\int_0^{\infty} e^{-x^2}\,dx \qquad I=\int_0^{\infty} e^{-x^2}\,dx$$

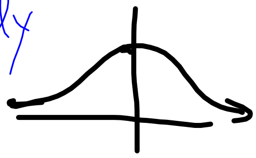
$$= 2\int_0^{\infty} e^{-y^2}\,dy$$

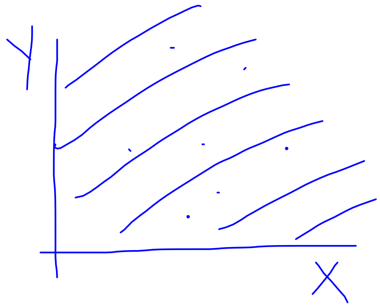
$\int_0^4$

$\int_0^1$ $\int_1^4$

$\int_0^5$

$$I^2 = \int_0^\infty e^{-x^2}dx \int_0^\infty e^{-y^2}dy$$





$$= \int_0^\infty\int_0^\infty e^{-x^2}e^{-y^2}\,dx\,dy$$

$$= \int_0^\infty\int_0^\infty e^{-x^2-y^2}\,dx\,dy$$

$$I^2 = \frac{\pi}{4}$$

$$I = \frac{\sqrt{\pi}}{2}$$

$$= \int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\,dx\,dy$$

$$= \left(\int_0^{\frac{\pi}{2}} d\theta\right)\left(\int_0^\infty e^{-r^2}\,r\,dr\right)$$

$$-\frac{1}{2}e^{-r^2} \qquad \cancel{-\frac{1}{2}} e^{-r^2}(-r^2)'$$

$$-2r$$

$$= \int_{\theta=0}^{\frac{\pi}{2}} \int_0^\infty e^{-r^2}\,r\,dr\,d\theta$$

$$\boxed{\begin{aligned} u &= -r^2 \\ du &= -2r\,dr \\ -\tfrac{1}{2}du &= r\,dr \\ &\int_0^{-\infty} -\tfrac{1}{2}e^{u}\,du \end{aligned}}$$

$$= \frac{\pi}{2} \cdot \left[-\frac{1}{2}e^{-r^2}\right]_0^\infty = \frac{\pi}{4}$$