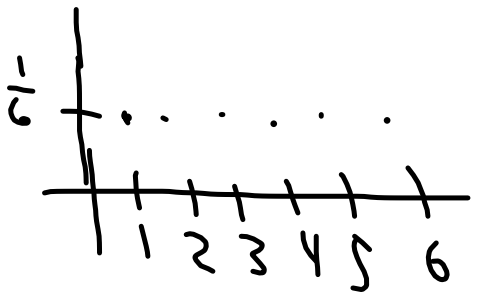
An experiment consists of rolling a normal six-sided die. Let $X$ be the random variable that assigns to each roll the number that lands up.

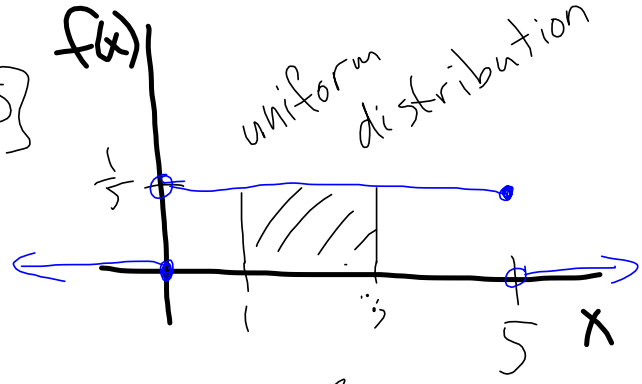
① What is Ran($X$)?

② Give the probability distribution function.

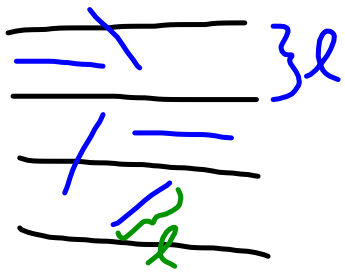
| $x$: | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $f(x)$: | $\frac{1}{6}$ | $\frac{1}{6}$ | - | - | - | - |



$$f(x) = \begin{cases} \frac{1}{5} & \text{for } x \in [0,5] \\ 0 & \text{otherwise} \end{cases}$$



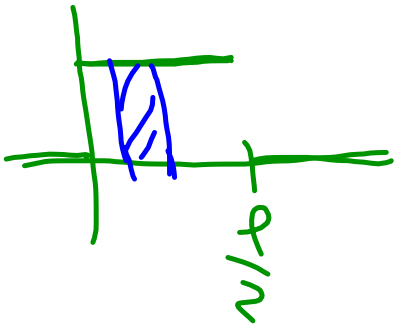

$$P(1 \le X \le 3) = \int_1^3 \frac{1}{5}\,dx = \frac{1}{5}x\Big|_1^3 = \frac{3}{5} - \frac{1}{5} = \frac{2}{5}$$

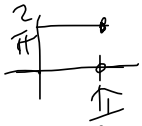



Experiment: Drop spag randomly.

$\bar{X}$ is the dist of the center of spag to nearest line

$\theta$ is the acute angle btwn lines and the extended spaghetti



$Ran(X) = [0, \frac{\ell}{2}]$ $Ran(\theta) = [0, \frac{\pi}{2}]$ 

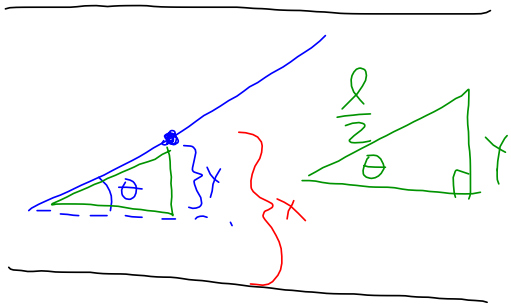

$$g(x) = \begin{cases} \frac{2}{\ell} \text{ if } x \in [0, \frac{\ell}{2}] \\ 0 \text{ otherwise} \end{cases} \qquad h(\theta) = \begin{cases} \frac{2}{\pi} \text{ for } \theta \in [0, \frac{\pi}{2}] \\ 0 \text{ otherwise} \end{cases}$$

Do you think $X$ and $\Theta$ are independent?

$$f(x,y) = g(x)h(y)$$

$$f(x,\theta) = g(x)h(\theta) = \frac{2}{\ell}\cdot\frac{2}{\pi} = \frac{4}{\pi\ell} \quad \text{on } \left[0,\frac{\ell}{2}\right]\times\left[0,\frac{\pi}{2}\right]$$

$$P\left(\begin{array}{c}\text{condition}\\ \text{for spag to}\\ \text{touch a line}\end{array}\right) = \int\int \frac{4}{\pi\ell}\,dx\,d\theta = \int\int \frac{4}{\pi\ell}\,d\theta\,dx$$



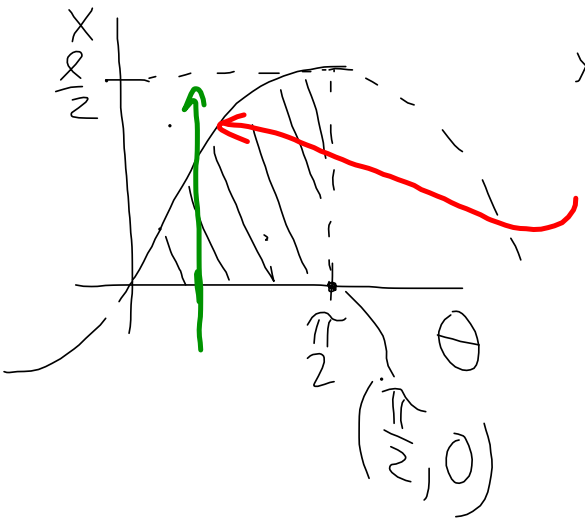

$\frac{y}{\frac{\ell}{2}} = \sin\theta$

$y = \frac{\ell}{2}\sin\theta$

$y \geq x \Rightarrow$ spag will touch

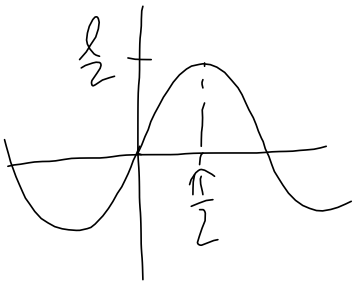
$y = \frac{\ell}{2}\sin x$

$\frac{\ell}{2}\sin\theta \geq x$




$x \leq \frac{\ell}{2} \sin\theta$

$x = \frac{\ell}{2} \sin\theta$



$$P(\text{spag touches}) = \int_{\theta=0}^{\frac{\pi}{2}} \int_{x=0}^{\frac{\ell}{2}\sin\theta} \frac{4}{\pi\ell}\, dx\, d\theta$$

$$= \frac{4}{\pi\ell} \int_0^{\frac{\pi}{2}} \left[x\right]_0^{\frac{\ell}{2}\sin\theta} d\theta$$

$$= \frac{4}{\pi\ell} \int_0^{\frac{\pi}{2}} \frac{\ell}{2}\sin\theta\, d\theta$$

$$= \frac{2}{\pi}\left[-\cos\theta\right]_0^{\frac{\pi}{2}} = \frac{2}{\pi}\left[-\cos\frac{\pi}{2} - (-\cos 0)\right] = \frac{2}{\pi}$$

Buffon's Needle Problem

For Wednesday

1. Find your three exams + bring them
2. Do 1.7: 3-6, check answers
3. Do course evaluation