$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x}\,dx$$

1. Write out and simplify (as much as you can) $\Gamma(\frac{1}{2})$
2. Make the substitution $x = u^2$ and simplify.

(3) Remembering that $\int_0^\infty e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$, what is $\Gamma(\frac{1}{2})$?

(4) Remembering that $\Gamma(\alpha) = (\alpha-1)\Gamma(\alpha-1)$, find $\Gamma(\frac{3}{2}), \Gamma(\frac{5}{2}), \Gamma(\frac{7}{2}), \Gamma(\frac{n}{2})$ for $n = 1, 2, 3, \ldots$

$$\Gamma\left(\frac{1}{2}\right) = \int_0^\infty x^{-\frac{1}{2}} e^{-x}\,dx \qquad \int_0^\infty e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}$$

$$= \int_0^\infty (u^2)^{-\frac{1}{2}} e^{-u^2}\, 2u\,du$$

Let $x = u^2$

$dx = 2u\,du$

When $x = 0$, $u = 0$

$x = \infty$, $u = \infty$

$$= 2\int \frac{1}{\cancel{u}} e^{-u^2} \cancel{u}\,du = \sqrt{\pi}$$

$$\Gamma\left(\frac{3}{2}\right) = \frac{\sqrt{\pi}}{2}$$

$$\Gamma\left(\frac{5}{2}\right) = \frac{3\sqrt{\pi}}{4}$$

$$\Gamma\left(\frac{7}{2}\right) = \frac{15\sqrt{\pi}}{8}$$

$$\Gamma\left(\frac{9}{2}\right) = \frac{105\sqrt{\pi}}{16}$$

$$\Gamma\left(\frac{n}{2}\right) = \frac{\sqrt{\pi}}{2^{\frac{n-1}{2}}}$$

n is odd

$2n$ $n = 1, 2, \ldots$ even

$2n+1$ $n = 1, 2, \ldots$

$$\Gamma\left(\frac{2n+1}{2}\right) = \frac{\overset{(2n-1)(2n-3)\cdots 1}{7 \cdot 5 \cdot 3 \cdot 1}\sqrt{\pi}}{2^n}$$

$$\Gamma\left(\frac{2n+1}{2}\right) = \frac{(2n-1)(2n-3)\cdots 1\sqrt{\pi}}{2^n}$$

$n = 4$

$2n+1 = 9$

$2n-1 = 7$

$$\overset{\text{odd}}{9!} = 9 \cdot 7 \cdot 5 \cdot 3 \cdot 1 = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot \cdots 1}{8 \cdot 6 \cdot 4 \cdot 2}$$

$$= \frac{9!}{2^4(4 \cdot 3 \cdot 2 \cdot 1)}$$

$$\overset{odd}{(2n+1)!!} = \frac{(2n+1)!}{2^n n!}$$

$n=4$

8 a) $P(X=1) = h(1;5,6,300) = 0.0934$

b) $P(X \geq 1) = 1 - H(0;5,6,300) = 0.0967$