Math 465 EXAM 1, WINTER 2017 **Name:** 74 points

- **Each numbered exercise is worth six points unless stated otherwise.**
- **DO NOT** reduce probabilities or give them as decimals, unless you are given decimal probabilities to begin with.
- **DO NOT** multiply out any answers obtained by products.
- In the extra spaces provided, give any trees or Venn diagrams used.

1. A card is drawn at random from a standard deck of 52 cards. Let Events A, B and C be as follows:

   - **Event C:** A club is drawn.
   - **Event T:** A two is drawn.
   - **Event F:** A face card (J, Q, K) is drawn.

   more space →

   Find each of the following probabilities. *2 points each*

   (a) $P(F) =$ $12/52 = 3/13$

   (b) $P(C \cup F) =$ $\frac{13+9}{52} = \frac{22}{52} = \frac{11}{26}$

   (c) $P(C \cap F) =$ $3/52$

   (d) $P(T \cap F) =$ 0

   (e) $P(C|F) =$ $3/12 = 1/4$

   (f) $P(F|C) =$ $3/13$

   } more space

2. Consider again the experiment and events from Exercise 1.

   (a) Circle all of the pairs of events that are independent. *3 points*

   (C and T) (C and F) T and F

   (b) Circle all of the pairs of events that are mutually exclusive. *3 points*

   C and T C and F (T and F)

10	0
9	2222168835
8	9119260759
7	93
6	38
5	86
4	9

+18

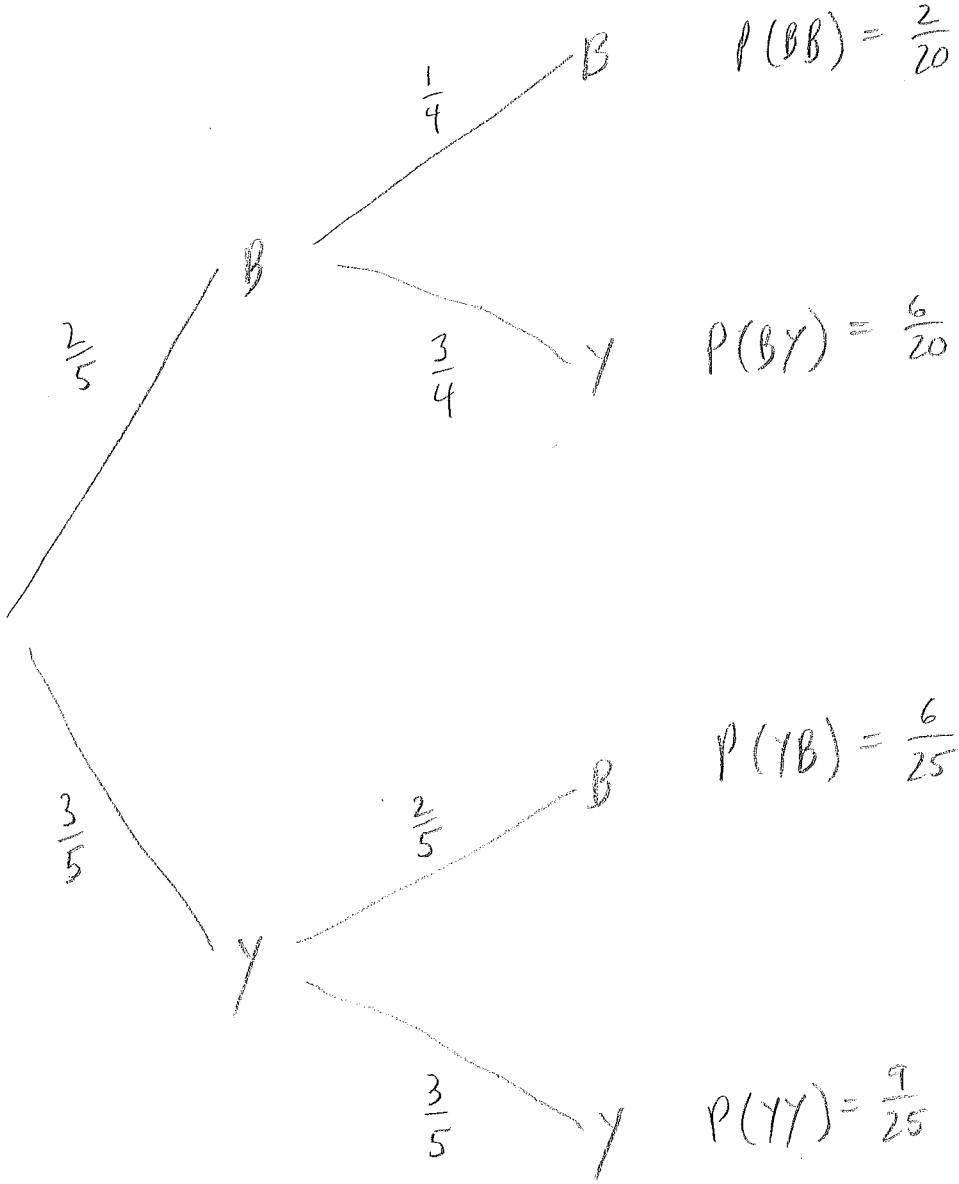
3. An urn contains two blue marbles and three yellow marbles. Consider the experiment of drawing two marbles, *replacing when a yellow is drawn and not replacing when a blue is drawn.* Find the probability of each of the following. *3 points each*

(a) drawing two yellow marbles: $\frac{9}{25}$

(b) drawing a yellow marble and a blue marble, *not necessarily in that order*: $\frac{6}{20} + \frac{6}{25} = \frac{54}{100} = \frac{27}{50}$

(c) getting a yellow marble on the second draw: $\frac{6}{20} + \frac{9}{25} = \frac{66}{100} = \frac{33}{50}$

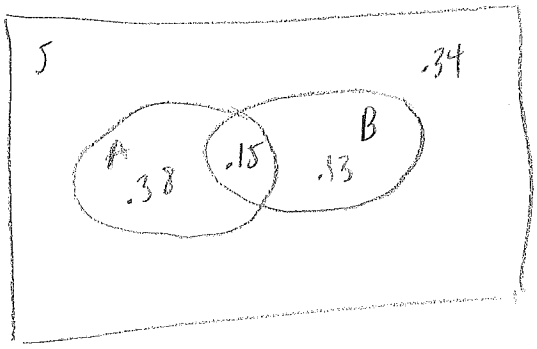
(d) getting a yellow marble on the second draw, *given that a blue marble was obtained on the first draw*: $\frac{3}{4}$



4. The probability of Event A is 0.53, the probability of Event B is 0.28, and the probability that both events occur $(A \cap B)$ is 0.15. Give each of the following probabilities:

$P(A \cup B) =$ 0.66 $\qquad$ $P(A|B) =$ $\frac{.15}{.28}$

The probability that neither A nor B occurs: 0.34



5. An experiment consists of drawing one marble from an urn containing five red marbles, eight blue marbles and two green marbles. The sample space for this experiment is $S = \{R, B, G\}$. We define a random variable $X : S \to \mathbb{R}$ by the number of letters in the spelling of the color that is drawn. So, for example, $X(R) = 3$.

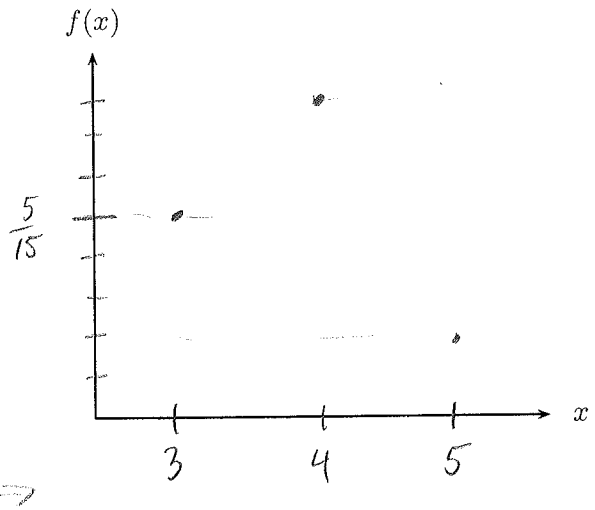
(a) Give the range of the random variable: $\text{Ran}(X) =$ $\{3,4,5\}$ *1 point*

(b) Give each of the following. *2 points each*

$P(X = 5) =$ $\frac{2}{15}$ $\qquad$ $P(X \geq 4) =$ $\frac{2}{15} + \frac{8}{10} = \frac{10}{15} = \frac{2}{3}$

(c) On the axes below and to the left, **graph** the probability distribution function f. Put an appropriate scale on each axis, and in such a way as to utilize most of the space available. *3 points*

less space →



$$F(x) = \begin{cases} 0 & \text{if } x < 3 \\ \frac{5}{15} & \text{if } 3 \leq x < 4 \\ \frac{13}{15} & \text{if } 4 \leq x < 5 \\ 1 & \text{if } x \geq 5 \end{cases}$$

more space →

(d) Give the cumulative distribution function F above and to the right. *Remember that it must be defined for all real numbers. 3 points*

+17

6. The range of a random variable is $\text{Ran}(X) = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Give each of the following in terms of f, *without using summation notation.*

(a) $P(X = 3) =$ $f(3)$

(b) $P(3 \leq X \leq 5) =$ $f(3)+f(4)+f(5)$

(c) $P(X \leq 3) =$ $f(1)+f(2)+f(3)$

(d) $P(X \geq 3) =$ $1 - f(2)$

7. The range of a random variable is $\text{Ran}(X) = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Give each of the following in terms of F, *without using summation notation.*

(a) $P(X = 3) =$ $F(3) - F(2)$

(b) $P(3 \leq X \leq 5) =$ $F(5) - F(2)$

(c) $P(X \leq 3) =$ $F(3)$

(d) $P(X \geq 3) =$ $1 - F(2)$

8. (a) Eight OIT students decide to form a club. The club needs a president, vice-president and secretary (no two of which can be the same person). How many ways could these be selected from all eight members? (The same three people but with different duties are counted separately.) **Give an expression that gives the correct answer - you do not actually have to compute the result.**

$8 \cdot 7 \cdot 6$

(b) Three members of the club are needed to staff a table at the Super Club Sign-Up. How many three member "teams" are possible using the eight members of the club, assuming that the people on a team do not have specialized duties? **Give an expression that gives the correct answer - you do not actually have to compute the result.**

$\frac{8!}{3!5!}$ or $\frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1}$

9. (a) How many ways can we line up 3 pennies, 2 nickels, and 5 dimes, if rearrangements within each denomination are not considered to be different?

$\frac{10!}{3!2!5!}$

(b) Jim is making up a password, to consist of the four digits 0, 1, 2, 3 (each used exactly once) followed by the letters of his name (each of these also used exactly once). He doesn't want the first digit to be zero, and he doesn't want the letters in his name to be in the order JIM. How many passwords are possible?

$(3 \cdot 3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1) - (3 \cdot 3 \cdot 2 \cdot 1 \cdot 1 \cdot 1 \cdot 1)$

$108 - 18 = 90$

[redacted]

+24