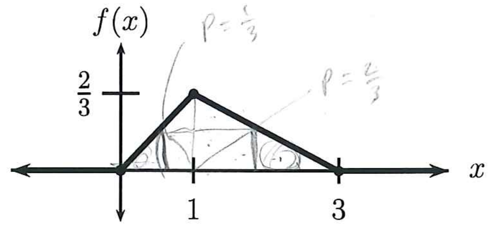
Math 465 EXAM 2, WINTER 2017 **Name:** 52 points

- Each numbered exercise is worth six points unless stated otherwise.
- When an integral is required to obtain an answer, **indicate clearly the integral used**. You may use your calculator to calculate the integral itself.

1. The probability density function, and its graph, for a *continuous* random variable X are shown below. Use them to find the things that follow, **as numbers. Optional:** Show any work you do in the space below for the possibility of partial credit for incorrect answers. *2 points each*

$$f(x) = \begin{cases} \frac{2}{3}x & \text{for } 0 \le x < 1 \\ 1 - \frac{1}{3}x & \text{for } 1 \le x \le 3 \\ 0 & \text{elsewhere} \end{cases}$$



(a) $P(X > 1) =$ $\frac{2}{3}$

(b) $P(X = 2) =$ 0

(c) $P(\frac{1}{2} < X < 2) =$ $\frac{3}{4}$

(d) $F(0) =$ 0

(e) $F(7) =$ 1

$$\int_{\frac{1}{2}}^{1} \frac{2}{3}x\,dx + \int_{1}^{2} (1-\tfrac{1}{3}x)\,dx = \tfrac{1}{3}x^2\Big|_{\frac{1}{2}}^{1} + \left[x - \tfrac{1}{6}x^2\right]_1^2 = \frac{1}{3} - \frac{1}{12} + (2-\tfrac{2}{3}) - (1-\tfrac{1}{6})$$

$$= \frac{4}{12} - \frac{1}{12} - \frac{8}{12} + \frac{2}{12} + 1 = \frac{3}{4}$$

$\frac{3}{12}, \frac{6}{12}, \frac{9}{12}$

2. Find the expected value (as a number) for the distribution from Exercise 1, **indicating clearly how you do it.**

$$E(X) = \int_0^1 \frac{2}{3}x^2\,dx + \int_1^3 (1-\tfrac{1}{3}x)x\,dx \quad +4$$

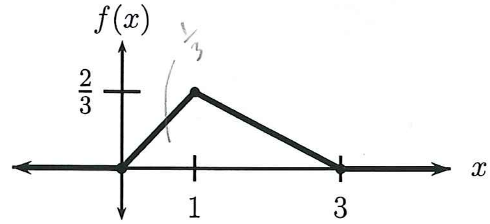
$$= \frac{2}{9}x^3\Big|_0^1 + \left[\tfrac{1}{2}x^2 - \tfrac{1}{9}x^3\right]_1^3$$

$$= \frac{2}{9} + \left[\frac{9}{2} - 3\right] - \left[\frac{1}{2} - \frac{1}{9}\right] = \frac{3}{9} + \frac{8}{2} - 3 = \boxed{1\tfrac{1}{3}} \quad +2$$

+16

3. The probability density function, and its graph, from Exercise 1 are shown below.

$$f(x) = \begin{cases} \frac{2}{3}x & \text{for } 0 \le x < 1 \\ 1 - \frac{1}{3}x & \text{for } 1 \le x \le 3 \\ 0 & \text{elsewhere} \end{cases}$$



Give the cumulative probability function $F(x)$ for the density function f. **Show work (in the space below and to the left or on scratch paper) if you want the chance of partial credit for an incorrect answer.**

$1 \le x \le 3$:

$F(x) = \frac{1}{3} + \int_1^x \left(1 - \frac{1}{3}t\right) dt$

$= \frac{1}{3} + \left[t - \frac{1}{6}t^2\right]_1^x$

$= \frac{1}{3} + x - \frac{1}{6}x^2 + 1 - \frac{1}{6}$

$$F(x) = \begin{cases} 0 \text{ for } x < 0 & +1 \\ \frac{1}{3}x^2 \text{ for } 0 \le x < 1 & +1 \\ -\frac{1}{6}x^2 + x + \cancel{\frac{7}{6}}\ {-\frac{1}{2}} \text{ for } 1 \le x \le 3 & \frac{1}{3} \\ 1 \text{ for } x > 3 & +1 \end{cases}$$

4. A discrete random variable has probability density function

x:	0	1	2	3
$f(x)$:	$\frac{27}{64}$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{1}{64}$

Find the variance σ^2 of this distribution, **indicating CLEARLY how you got it.**

27
18
3
48

$E(X) = \frac{27}{64} + \frac{18}{64} + \frac{3}{64} = \frac{48}{64} = \frac{3}{4}$

$E(X^2) = \frac{27}{64} + \frac{36}{64} + \frac{9}{64} = \frac{72}{64} = \frac{9}{8}$

$\sigma^2 = E(X^2) - [E(X)]^2 = \frac{9}{8} - \left(\frac{3}{4}\right)^2 = \frac{18}{16} - \frac{9}{16} = \boxed{\frac{9}{16}} \approx 0.563$

+12

For exercises 5 through 7,

- Give a probability statement $P($ *something* $)$, followed by (with an equal sign!) an expression containing the numerical values of all variables and parameters, in terms of the appropriate probability function or cumulative probability function that would give the desired probability. **Use the cumulative function if the probability of more than one value is sought.**
- If the normal distribution is used, find a numerical value for the answer in addition to the above.

5. An opaque Safeway grocery bag contains 14 caramels and 9 chocolates, all of which are the same size and shape. You randomly select 5 candies to bring to school. What is the probability that exactly 2, 3 or 4 of them are caramels?

$$P(2 \le X \le 4) = \cancel{H(4;5,9,14) - H(1;5,9,14)}$$

+1

$$= H(4;5,14,23) - H(1;5,14,23)$$

$+4\frac{1}{2}$ for Eh

$+4\frac{1}{2}$ if 2

+2 for $B(4;5,\frac{14}{23}) - B(1;5,\frac{14}{23})$

6. A machine makes electrical resistors having a mean resistance of 40 ohms and a standard deviation of 2 ohms. Assume that the resistance follows a normal distribution and can be measured to any degree of accuracy. What is the probability of randomly selecting a resistor with a resistance greater than 39 ohms?

$$P(X \ge 39) = 1 - N(39;40,2) = 1 - N(-0.5;0,1)$$

+1 +1 +3

$$= 1 - 0.3085$$

$$= \boxed{0.6915}$$ +1

1.0000
.3085
$\boxed{.6915}$

7. Consider the same opaque Safeway grocery bag containing 14 caramels and 9 chocolates, but suppose instead of the above that you decide you want a chocolate, so randomly start selecting candies from the bag. It is early in the morning (10:00 AM) and you aren't thinking thinking clearly, so after each selection of a caramel you return it to the bag. Assuming that any returned candies assimilate themselves with the others, what is the probability that you don't obtain a chocolate until the fifth draw?

$$P(X = 5) = b^*(5;1,\tfrac{9}{\cancel{14}\ 23})$$

+1 +2 $+\frac{1}{2}$ $+1\frac{1}{2}$ +1

+18

Do **EXACTLY TWO** of the following exercise in the space below. **Cross out the one that you don't want me to grade.**

8. An urn contains 4 red marbles and 6 green marbles. Marbles are drawn *without replacement* from the urn until two red marbles have been obtained. What is the probability that it takes exactly ~~nine~~ five draws for this to happen? *You cannot use any of the standard distributions to solve this problem, to my knowledge.*

9. Give the value of C for which the function f given to the right is a continuous probability density function.
$$f(x) = \begin{cases} Cx^2 & \text{for } 0 \le x < 1 \\ 2C - Cx & \text{for } 1 \le x < 2 \\ 0 & \text{elsewhere} \end{cases}$$

10. Lengths of screws are normally distributed, with mean 2.31 cm and standard deviation 0.03 cm. Assuming that selecting a screw is a Bernoulli process, find the probability that 8 *or fewer* screws must be selected to obtain five screws screws less than 2.35 cm long. **Do not actually find the answer, but indicate which distribution(s) to use and how (values of all variables and parameters).**

(8) $\boxed{8\left[\left(\frac{2}{5}\right)^2\left(\frac{3}{5}\right)^3\right]}$ $\boxed{4\left(\cancel{\frac{2^2\,3^3}{10\cdot 9\cdot 8\cdot 7\cdot 6}}\right)}$

+3 +3

$4\left(\frac{4\cdot 3\cdot 6\cdot 5\cdot 4}{10\cdot 9\cdot 8\cdot 7\cdot 6}\right)$

(9) $\int_0^1 Cx^2\,dx + \int_1^2 (2C - Cx)\,dx = \frac{C}{3}x^3\Big|_0^1 + \left[2Cx - \frac{C}{2}x^2\right]_1^2$

+2

$= \frac{C}{3} + (4C - 2C) - (2C - \frac{C}{2})$

+3

+1 idea

$= \frac{C}{3} + \frac{C}{2} = \frac{5C}{6} = 1$ $\boxed{C = \frac{6}{5}}$

(10) $y = P(X \le 2.35) = N(2.35; 2.31, 0.03) = N(1.33; 0, 1) = 0.9082$

$P(X \le 8) = B^*(8; 5, 0.9082)$

+1 cum

+1 all parameters + variables

+2 neg binom

+2 idea

#8 $4\left(\frac{4\cdot 3\cdot 6\cdot 5\cdot 4}{10\cdot 9\cdot 8\cdot 7\cdot 6}\right)$

$\frac{4\cdot 4\cdot 3\cdot 6\cdot 5\cdot 4}{10\cdot 9\cdot 8\cdot 7\cdot 6}$ $\frac{4}{21}$

16