Math 465 EXAM 3, WINTER 2017 **Name:** 87 points (or 81)

Each numbered exercise is worth six points unless stated otherwise.

1. The following table gives the joint distribution for two discrete random variables X and Y.

		x			
	$f(x,y)$	1	2	3	
y	0	$\frac{5}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{12}{16}$
	1	$\frac{2}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	
			$\frac{5}{16}$		

Use the above table for each of the following. *3 points each*

(a) $P(X = 2 \text{ or } Y = 0) =$ $\frac{5}{16} + \frac{4}{16} + \frac{3}{16} + \frac{1}{16} = \frac{13}{16}$

(b) $P(X = 2, Y = 0) =$ $\frac{4}{16}$

(c) $P(Y \geq 3 - X) =$ $\frac{3}{16} + \frac{1}{16} + \frac{1}{16} = \frac{5}{16}$

(d) $P(X = 3 \mid Y = 1) =$ $\frac{\frac{1}{16}}{\frac{4}{16}} = \frac{1}{4}$

(e) Give $v(x \mid 0)$ using some appropriate format.

$x:$	1	2	3
$v(x\mid 0):$	$\frac{5}{12}$	$\frac{4}{12}$	$\frac{3}{12}$

$\frac{3}{16}$
16
96
160
256

2. Are X and Y independent? Circle your answer: yes no **Show work below that supports your answer.** *3 points*

$g(1)h(0) = \frac{7}{16} \cdot \frac{12}{16} = \frac{84}{256} \neq \frac{80}{256} = f(1,0)$

+18

3. The following table gives the joint distribution for two discrete random variables X and Y.

		x			
	$f(x,y)$	1	2	3	
y	0	$\frac{5}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{12}{16}$
	1	$\frac{2}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{4}{16}$
		$\frac{7}{16}$	$\frac{5}{16}$	$\frac{4}{16}$	

Use the above table for each of the following. *3 points each*

(a) Give $h(y)$ using some appropriate format.

y:	0	1
$h(y)$:	$\frac{12}{16}$	$\frac{4}{16}$

(b) $E(X) = 1\left(\frac{7}{16}\right) + 2\left(\frac{5}{16}\right) + 3\left(\frac{4}{16}\right) = \frac{7}{16} + \frac{10}{16} + \frac{12}{16} = \boxed{\frac{29}{16}}$

$E(Y) = \frac{4}{16} = \frac{1}{4}$

(c) $E(XY) = 1\left(\frac{2}{16}\right) + 2\left(\frac{1}{16}\right) + 3\left(\frac{1}{16}\right) = \frac{2}{16} + \frac{2}{16} + \frac{3}{16} = \boxed{\frac{7}{16}}$

(d) $\sigma_{XY} = E(XY) - E(X)E(Y) = \frac{7}{16} - \frac{29}{16} \cdot \frac{1}{4} = \frac{28}{64} - \frac{29}{64} = \boxed{-\frac{1}{64}}$

+12

4. One card is selected from a standard deck of cards. Let X be the random variable that assigns the number of fives drawn and let Y be the random variable that assigns the number of hearts drawn. In the space to the right, construct a table for the joint probability distribution $f(x,y)$.

		X (fives)	
	$f(x,y)$	0	1
Y (hearts)	0	$\frac{36}{52}$	$\frac{3}{52}$
	1	$\frac{12}{52}$	$\frac{1}{52}$

+1 each entry
+2 table form

$52 - 16 = 36$

5. Use the distributions below for this exercise. *3 points*

Distribution 1

		x		
	$f(x,y)$	0	1	
y	0	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{1}{2}$
	1	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{1}{2}$
		$\frac{1}{2}$	$\frac{1}{2}$	

$\frac{2}{6} - \frac{1}{4}$

$\frac{4}{12} - \frac{3}{12} = \frac{1}{12} = \sigma_{xy}$

Distribution 2

$\sigma_{xy} = \frac{1}{2} - \frac{11}{16} \cdot \frac{12}{16} = \frac{128}{256} - \frac{132}{256} = -\frac{4}{256} = -\frac{1}{64}$

		x		
	$f(x,y)$	0	1	
y	0	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{4}{16}$
	1	$\frac{4}{16}$	$\frac{8}{16}$	$\frac{12}{16}$
		$\frac{5}{16}$	$\frac{11}{16}$	

Distribution 3

$\sigma_{xy} = \frac{2}{16} - \frac{1}{2} \cdot \frac{9}{16} = \frac{4}{32} - \frac{9}{32} = -\frac{5}{32}$

		x		
	$f(x,y)$	0	1	
y	0	$\frac{1}{16}$	$\frac{7}{16}$	$\frac{1}{2}$
	1	$\frac{6}{16}$	$\frac{2}{16}$	$\frac{1}{2}$
		$\frac{7}{16}$	$\frac{9}{16}$	

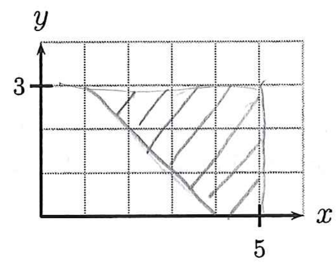
(a) Which distribution would have covariance closest to zero? (circle one number) 1 2 3

(b) Which distribution would have a negative covariance? (circle one number) 1 2 3

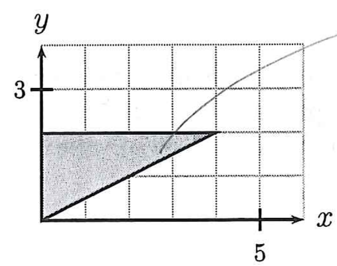
(c) Which distribution would have a positive covariance? (circle one number) 1 2 3

6. A continuous joint probability distribution $f(x,y)$ is nonzero only on the region $[0,5] \times [0,3]$ in the xy-plane. On the grid to the right, outline and shade the region over which we would need to integrate the nonzero part of $f(x,y)$ to find $P(X+Y \geq 4)$.

$Y = 4 - x$



7. Set up **ONE** iterated integral to integrate the function $f(x,y)$ over the region shown to the right.



$y = \frac{1}{2}x$

$x = 2y$

order +2

+2

$$\int_0^4 \int_{\frac{1}{2}x}^{2} f(x,y)\,dy\,dx = \int_0^2 \int_0^{2y} f(x,y)\,dx\,dy$$

+1

order +1

10

+2

+2½ outer & order of dxdy

+2½ inner

No dydx −2½

+21

For each of the following, give the name of the distribution used (*1 point*), a P(*something about* X) statement (*1 point*), and an expression using the appropriate distribution will all variables and parameters filled in that would give the desired probability (*4 points*).

8. Waterman likes to eat "cuties" this time of year. Past experience tells him that 16 cuties in a bag of 20 are generally really good. He opens a new bag of cuties and eats five of them (without replacement, of course!). What is the probability that exactly four of them are good?

hypergeometric, $P(X=4) = h(4; 5, 16, 20)$

9. On a toll road in Colorado a camera takes a photograph of your license plate as you pass by a particular point, and a toll bill is mailed to your home address (this is true, I can attest to it). Assume that during non-rush hours cars pass by the camera at a rate of 17 per minute. Find the probability that 80 to 100 cars (including both of those values) will pass by the camera during a five minute non-rush hour period.

Poisson, $P(80 \le X \le 100) = P(100; (17)(5)) - P(79; (17,5))$

+1 +1 +1 +1

10. A basketball player's free throw percentage is 0.65. Suppose that the player finishes each practice by shooting free throws until they make 20 of them. Assuming that they get no better while practicing, what is the probability that it will take them ~~between~~ 25 or more attempts to finish their practice?

negative binomial, $P(X \ge 25) = 1 - B^*(24; 20, 0.65)$

11. A coin is flipped 12 times. What is the probability that this will result in somewhere between 4 and 7 heads (including both of those values) will be obtained?

binomial, $P(4 \le X \le 7) = B(7; 12, \frac{1}{2}) - B(3; 12, \frac{1}{2})$

12. Rock climbing ropes have an inner core that provides their strength, surrounded by a woven sheath that protects the core. If the sheath has an average of 0.05 flaws per foot, what is the probability of finding a section of rope 30 feet or shorter with no flaws?

20 ft/flaw

exponential, $P(X \le 30) = F(30) = 1 - e^{-\frac{20}{30}} = 0.4866$

$1 - (1 - P(0; 1.5))$

$P(0; (30)(.05)) =$

$\times$ 30